ADAPTIVEDYNAMICS
Exercises 4-7

Exercise 4 (prey evolution)
Consider the resident dynamics for different prey strategies and a single (non-evolving) predator:

$$\begin{aligned} prey: \quad \frac{\dot{n}_i}{n_i} &= r(x_i)\left(1 - \frac{\sum_j n_j}{K}\right) - \frac{\beta(x_i)\theta}{1+\sum_j \beta(x_j)T(x_j)n_j} \qquad i = 1, \ldots, k \\ pred: \quad \frac{\dot{\theta}}{\theta} &= \frac{\sum_j \gamma(x_j)\beta(x_i)n_j}{1+\sum_j \beta(x_j)T(x_j)n_j} - \delta \end{aligned} \tag{1}$$

(a) Write down the invader dynamics for a prey mutant with strategy y.
(b) Write down an expression for the invasion fitness.
(c) Show that coexistence of different resident strategies is generically not possible.
(d) Show that y can invade x if $\beta(y)/r(y) < \beta(x)/r(x)$.
(e) Do the dynamics of θ really matter?

Exercise 5 (predator evolution)
Consider the resident dynamics for different predator strategies and a single (non-evolving) prey:

$$\begin{aligned} prey: \quad \frac{\dot{\theta}}{\theta} &= r\left(1 - \frac{\theta}{K}\right) - \sum_j \frac{\beta(x_i)n_j}{1+\beta(x_j)T(x_j)\theta} \\ pred: \quad \frac{\dot{n}_i}{n_i} &= \frac{\gamma(x_i)\beta(x_i)\theta}{1+\beta(x_i)T(x_i)\theta} - \delta(x_i) \qquad i = 1, \ldots, k \end{aligned} \tag{2}$$

(a) Write down the invader dynamics for a predator mutant with strategy y.
(b) Write down an expression for the invasion fitness.
(c) How many different resident types could generically coexist if the resident population densities were constant (i.e., at an equilibrium) and how many if it were non-constant (e.g., at a limit cycle)? (The system is known to be able to exhibit limit cycles, by the way.)
(d) Show that in a constant monomorphic resident population successive invasions decrease the prey population density (i.e., in a constant population evolution minimizes prey population density).

Exercise 6 (evolution of virulence: model 1)
Consider the resident dynamics for healthy (H) individuals and individuals (I_i) infected by different types of virus:

$$\begin{aligned} healthy: \quad \dot{H} &= \alpha(N) - H\sum_j \beta(x_j)I_j - \delta H \\ Infected: \quad \dot{I}_i &= \beta(x_i)HI_i - (\delta + x_i)I_i \qquad i = 1, \ldots, k \end{aligned} \tag{3}$$

where $N = H + \sum_j I_j$ is the total population density.
(a) Give the invader dynamics of a mutant virus with strategy y.

(b) Give the invasion fitness.
(c) How many different types of virus can coexist in the population?
(d) Show that each successful invasion decreases the population density of healthy individuals.

Exercise 7 (evolution of virulence: model 2)
Consider the resident dynamics for healthy (H) individuals and individuals (I_i) infected by different types of virus:

$$\begin{aligned} healthy: \quad \dot{H} &= \alpha N - H \textstyle\sum_j \beta(x_j) I_j - \delta(N) H \\ Infected: \quad \dot{I}_i &= \beta(x_i) H I_i - (\delta(N) + x_i) I_i \qquad i = 1, \ldots, k \end{aligned} \tag{4}$$

where $N = H + \sum_j I_j$ is the total population density.
(a) What is the difference between this model and that of the previous exercise? **(b)** Give the invader dynamics of a mutant virus with strategy y.
(c) Give the invasion fitness.
(d) How many different types of virus can coexist in the population?
(e) Give an explicit expression of the invasion fitness in terms of the invader and resident strategies if the resident population contains two coexisting viral strategies x_1 and x_2.