IV Evolution in a dimorphic resident population

(→ Geritz et al 1998)

(Readily generalizes to polymorphic population.)

Low mutation rate
("mutation-limited evolution")

⇒ Only one mutant at a time

So, we can study the possible evolutionary change for each resident strategy separately, pretending the other resident strategy is fixed

This is how that goes:

First, let $\mathbb{P}_2$ denote the set of all strategy pairs (x_1, x_2) that can coexist

(Note that $\mathbb{P}_2$ is not necessarily the same as the set of mutually invading strategies, although in L.V. models this happens to be true)

Evolutionary isoclines.

The x_i-isocline ($i=1,2$) is the set

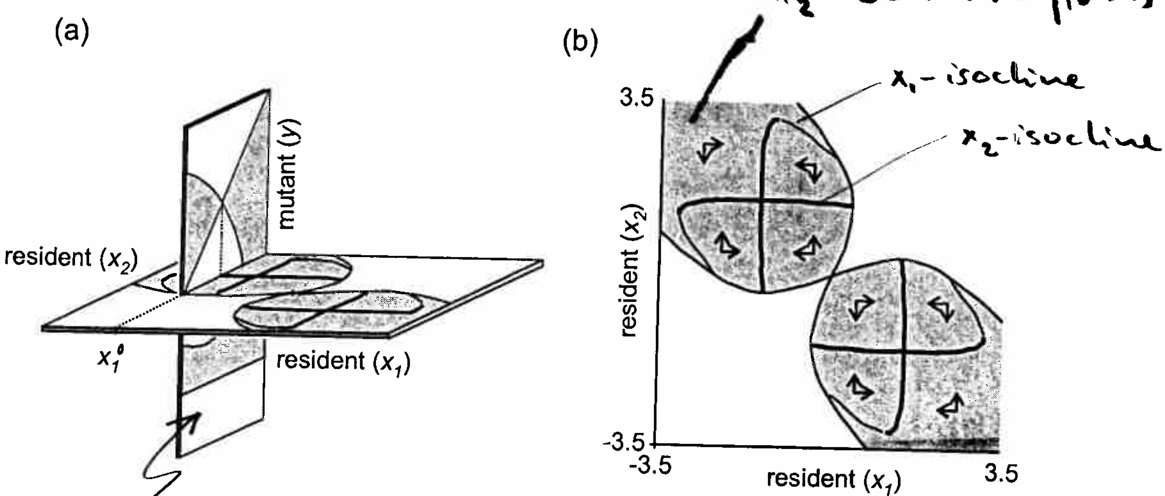
$$\{ (x_1,x_2) \in \mathbb{P}_2 \mid [\partial_y s_{x_1,x_2}(y)]_{y=x_i} = 0 \}.$$

In other words, the x_i-isocline is the collection of all points (x_1,x_2) where the selection gradient $\partial_y s_{x_1,x_2}(y)|_{y=x_i}$ (in the x_i-direction) vanishes.

In different words again, (x_1,x_2) lies on the x_i-isocline if x_i is singular with respect to evolution in x_i if x_j ($j \neq i$) were kept constant.

Example.

$\mathbb{P}_2$ (dark region)



PIP for x_2 assuming that x_1 stays fixed

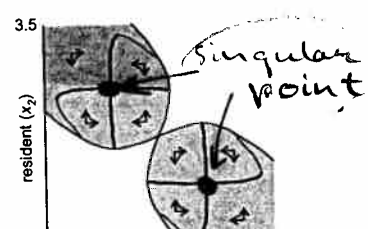
If the points on an isocline can be interpreted as "singular points" (in at least one of the strategies) they can be classified in a similar way (see "8 cases"):

Suppose $\partial_y S_{x_1,x_2}(y)\big|_{y=x_i} = 0$, i.e., (x_1,x_2) is a point on the x_i-isocline.

$\partial_y^2 S_{x_1,x_2}(y)\big|_{y=x_i} < 0$ $\Rightarrow$ (x_1,x_2) is uninvadable for (small) mutations in x_i (but not necessarily in x_j, $j \neq i$).

$\partial_y^2 S_{x_1,x_2}(y)\big|_{y=x_i} < \partial_{x_i}^2 S_{x_1,x_2}(y)\big|_{y=x_i}$

$\Rightarrow$ (x_1,x_2) is attracting for evolution in the x_i-direction (but not necessarily in the x_j ($j \neq i$) direction)



etc

If (x_1,x_2) lies both on the x_1-isocline and on the x_2-isocline, the we call (x_1,x_2) a singular point