

8.1 Exercises

1. Compute with WinBUGS a 'prediction' of CO₂-level (see lecture notes) for September 1968. (The true value was 320.41). What can we say about the linear model? Replace the linear model by adding exponential term (" $\alpha \exp(\beta t)$ "). Is the prediction better?

2. Compute the 'rats' example in WinBUGS manual. Compute a posterior prediction for a new rat at all 5 time points. Remove some rat specific measurements (replace them with 'NA' in the data). Compute posterior distribution for these 'missing data' and check how well the model predicted NA's. Describe (the idea of) how you would construct a cross validation assessment of the model, for comparing different models. Think also computational efforts: in full scale (cross validation with every single data point) this could become heavy (especially with larger data), but could you suggest a computationally lighter approach?

3. Compute the kangaroo example with WinBUGS using 2D-measurements, by specifying bivariate normal densities with unknown mean vectors, and unknown covariance matrices, parameterized using correlations ρ and standard deviations σ_x , σ_y , all with uniform priors. Compute the probability of male kangaroo for the measurements (1200, 750), (1450, 850), (1500, 900), (1800, 1000) and (1400, 750).

```
list(x=structure(.Data=c(
1439, 824, 1,
1413, 823, 1,
1490, 897, 1,
1612, 921, 1,
1388, 805, 1,
1840, 984, 1,
1294, 780, 1,
1740, 977, 1,
1768, 968, 1,
1604, 880, 1,
1464, 848, 2,
1262, 760, 2,
1112, 702, 2,
1414, 853, 2,
1427, 823, 2,
1423, 839, 2,
1462, 873, 2,
1440, 832, 2,
1570, 894, 2,
1558, 908, 2).Dim=(20,3)))
```

4. Check the sensitivity of the results of the meta analysis example (vitamin supplements) under different priors with WinBUGS, such as $\tau \sim \text{Gamma}(0.1, 0.1)$ and $\sigma \sim \text{U}(0, 1000)$. Would the essential conclusion of the meta analysis be different or just modestly different?

5. Consider the normal model with fixed variance, unknown mean. Use the same data as in the lecture notes, X_1, \dots, X_{10} :

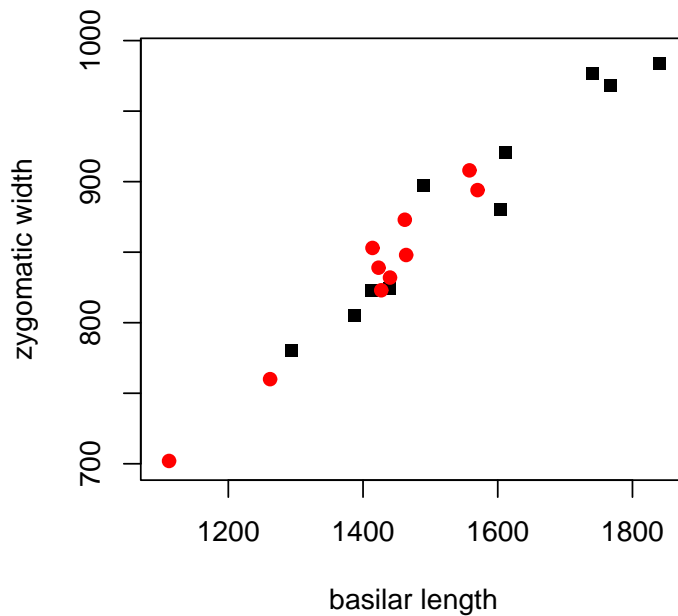


Figure 1: Kangaroo skull measurements. Red dots = female, black squares = male.

$x=c(-0.7417224, -2.1873614, 1.1508363, 0.1306749, -1.1931158, 0.2093445, -0.1040642, 1.230186, 0.910799, 0.1830353)$

Assume 3 different models with different σ -parameters. Assess model fitness with WinBUGS, or R, by computing bayesian p-value using all data (X_1, \dots, X_{10}) , and then using only smaller part of data. What happens to the p-values when the amount of data becomes smaller?

6. Consider the same normal model as above and compute with WinBUGS the bayesian p-value based on $T(X, \theta) = |X_{\text{smallest}} - \theta|$. Plot the scatter plot of $(T(X^*, \theta), T(X, \theta))$. (WB Menu: Inference \rightarrow correlations).

7. Assume the same series of 20 Bernoulli trials as in the lecture notes. Write a WinBUGS code and compute $P(T(x^{\text{rep}}) > T(x^{\text{obs}}) | x^{\text{obs}})$, where $T(\cdot)$ is the number of switches, $0 \rightarrow 1$, or $1 \rightarrow 0$, over the observed series.

8. Compute the normal hierarchical model below and compute the DIC in WinBUGS. (First run some iterations, then click 'inference' and 'DIC'). In this DIC assessment, the focus is on the conditional model of y given group specific parameters θ . Check the effective number of parameters, p_D . Does it make sense? How is this related to the example in lecture notes where all schools were analyzed separately? Replace the prior of `sigma.theta` by setting a small fixed value for `sigma.theta`. Compute DIC again and check the effective number of parameters p_D , explaining what happened. Which model is better in terms of DIC? You could also try developing the model in some ways, trying to achieve even better model fit in terms of DIC.

```

model{
for(i in 1:4){
for(j in 1:5){
y[i,j] ~ dnorm(theta[i],1)
}
theta[i] ~ dnorm(mu.theta,tau.theta)
}
mu.theta ~ dnorm(0,1.0E-6)
tau.theta <- pow(sigma.theta,-2)
sigma.theta ~ dunif(0,10000) # or replace this with: <- 0.1
}
list(
y=structure(.Data=c(
6.30, 5.99, 4.85, 3.60, 5.71,
0.82, 2.45, 2.34, 3.52, 2.42,
8.60, 9.63, 8.24, 9.23, 9.20,
2.58, 2.27, 3.68, 0.71, 3.01),.Dim=c(4,5)))

```