6.1 Exercises

1. Compute the approximate value of $\pi = 3.1415\ldots$ using the method previously described in WinBUGS. How many iterations are needed to get the first 4 decimals correct?

2. Assume the model $X \sim \text{Bin}(N, 0.2)$, and that $X = 1$ was observed. Compute the posterior distribution of N in WinBUGS. Try different priors for N.

3. According to an expert, the sensitivity of a testing method is roughly in the range of $0.3 - 0.7$. Construct a beta prior distribution in WinBUGS that is approximately in this range.

4. The observed life times were

```
X=c(1.54, 0.70, 1.23, 0.82, 0.99, 1.33, 0.38, 0.99, 1.97, 1.10,
0.40)
```

and there were 4 censored observations at time $T = 2$. Assume $X_i \sim \text{Exp}(\theta)$ and prior $\theta \sim \text{Gamma}(2, 1)$. Write a WinBUGS model and compute the posterior of θ. Compute also the predictive distribution for a new observation, X^*.

5. Normal model with unknown mean μ, known variance σ^2. Compute in WinBUGS the posterior of μ assuming either small or large σ. Compare the results if the prior is either `mu` $\sim$ `dnorm(0,0.0001)` or `mu` $\sim$ `dflat()`.

```
x=c(-0.7417224,-2.1873614,1.1508363,0.1306749,-1.1931158,0.2093445,-0.1040642)
```

6. Microbial samples are collected from individual animals and analyzed as pooled samples of 3 subsamples. (3 animal specific samples put together). Each pooled sample then results to either negative or positive test. A positive result is obtained if any of the 3 sub-samples were colonized. A negative result is obtained if none of the 3 sub-samples were colonized. Assume 50 pooled samples were analyzed and 1 was positive. Compute the posterior of pooled sample population prevalence, and the posterior of sub-sample (animal) population prevalence, using WinBUGS. Uninformative prior of animal prevalence can be used. What is the posterior probability that animal prevalence is $>$1%?

7. Proportion p of meals provided by a catering service are contaminated. Proportion q of consumed contaminated meals leads to illness. Assume uniform priors for (p, q). 300 meals were served in a conference and 5 people got sick. Compute in WinBUGS the posterior distribution of p assuming (1) $q = 0.5$ is known and assuming (2) q is unknown. When both parameters are unknown, plot their joint distribution. Extend the model with the unknown number of contaminated servings, Z, actually served. Compute the posterior predictive distribution for the number of illnesses in another conference with 100 people who were served the same food. Extend the model with experimental data in which 10 volunteers consumed contaminated meals and 2 of them got sick.

8. Consumers of broiler legs were given data loggers which measured the actual cooking time, t, and temperature, T, in the oven. The data show modestly negative correlation between t and T. Explain why this could be so. Compute the posterior distribution in WinBUGS assuming the following model for the logarithms. Compute also predictive distribution for a 'next' consumer. What is the probability

that he/she will cook (A) less than 50 minutes, (B) over 50 min, but $T > 175$, (C) over 50 min, but $T \in [135, 175]$, (C) over 50 min, but $T < 135$? Is the model prediction realistic?

$$\begin{bmatrix} \log(t) \\ \log(T) \end{bmatrix} \sim \mathrm{N}\left(\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \begin{bmatrix} \sigma_{11}^2 & \rho\, \sigma_{11}\sigma_{22} \\ \rho\, \sigma_{11}\sigma_{22} & \sigma_{22}^2 \end{bmatrix} \right)$$

$$\mu_i \sim \mathrm{N}(0, 0.001) \qquad \sigma_{ii}^2 \sim \mathrm{Gamma}(0.01, 0.01) \qquad \rho \sim \mathrm{U}(-1, 1)$$

Hint: use `inverse()` for computing the inverse covariance matrix in WinBUGS.

```
list(N=19, timetemp=structure(.Data=c(
    43,   179,
    44,   217,
    47,   206,
    49,   185,
    49,   166,
    53,   193,
    53,   180,
    56,   176,
    58,   167,
    59,   180,
    61,   132,
    62,   152,
    62,   136,
    72,   178,
    73,   168,
    78,   149,
    84,   169,
    99,   161,
   105,   148),.Dim=c(19,2)))
```

9. In 2001, 1962 Finnish voters were asked for their favorite political party. The result was

SDP	Kesk	Kok	Vihr	Vas	muu	yht
471	453	396	243	177	222	1962
24.0	23.1	20.2	12.4	9.0	11.2	100%

Using Dir(1,1,1,1,1,1)-prior and WinBUGS, compute the posterior density of the true population percentage for voters of each party. What is the probability that SDP was more popular than Keskusta?

10. Based on the results of the first 7 competitions of Ahonen and Janda, and uninformative prior, compute using WinBUGS the posterior probability $P(\mu_1 > \mu_2 \mid \text{data})$. Then, compute posterior predictive distribution for the result of the last competition for both jumpers. Knowing the total result of the first 3 competitions of the Four Hills Tournament, and results of 4 previous competitions too, what is the predictive probability that after the last competition, the difference of total points of the Tournament will be less than one point? Try predicting the whole Tournament, based on the four

pre-tournament competitions only.

11. Coal mine accidents in Britain, 1851-1962. Expand the model from the lecture notes by using two or three change points. Compute the results in WinBUGS and use WinBUGS graphical tools to display them.