5.1 Exercises

1. Using the inverse cumulative distribution method, generate Monte Carlo simulations of the exponential density $\pi(x) = \lambda \exp(-\lambda x)$.

2. Sample size calculation by simulation in R: assume prior uncertainty about population prevalence is described by $p_0 \sim \mathrm{U}(0,1)$. Sample of size N is to be collected, resulting to observation X that has conditional distribution $X \sim \mathrm{Bin}(N, p_0)$. From this random data, posterior density is to be computed $p \sim \mathrm{Beta}(X+1, N-X+1)$, so that this enables comparison of posterior estimates with the 'true p_0' that was drawn from the uniform distribution. Evaluate the predictive probability $P(\mid p_0 - p \mid > 0.1)$ by simulation: (1) draw p_0 from the prior, and then (2) X, given p_0, and (3) p, given X, (4) repeat M times. How big sample N is needed to have $P(\mid p_0 - p \mid > 0.1) < 0.05$?

3. Predicted values of some X from a distribution $\pi(X \mid \theta)$ are generated. Uncertainty about θ is accommodated by sampling the values of θ from some $\pi(\theta)$. Discuss how the prediction uncertainty might be described if we were not allowed to use probability density for θ? (i.e. if using non-bayesian methods where θ is unknown constant so that there is no distribution for it).

4. Implement (in R, or perhaps even in WinBUGS!) the Gibbs sampler for the simple 2D normal density with mean vector $(0,0)$ and with some correlation $-1 < \rho < 1$. How quickly you get a representative sample, if ρ is nearly ± 1, and if the starting value is 'in the corner'.

5. Assume 2D-normal model with mean (μ_1, μ_2) and covariance matrix

$$\begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}.$$

Solve the expressions of full conditional densities for a Gibbs sampler.

6. Continue problem 7 in exercise 3.1. Compute by simulation (e.g. in R) the posterior probability (based on all data) that μ_1 (Ahonen) is larger than μ_2 (Janda).

7. Simulate in R (in WinBUGS some numerical problems can happen) posterior density of the mean concentration of virus per litre, λ. (See the example in lecture notes). Simulate also the posterior of proportion of servings that have infective dose. What is the posterior probability of infection?

8. Simulate X from beta-binomial distribution, by using Gibbs sampler. (step 1): $X \sim \mathrm{Bin}(N, r)$, (step 2): $r \sim \mathrm{Beta}(X+\alpha, N-X+\beta)$. You can implement this in R, WinBUGS, or any suitable software.