

## 2.1 Exercises

1. Explain the role of uncertainty (epistemic uncertainty) and variability (aleatory uncertainty) in the context of bayesian inference by using some example (other than the lecture examples).
2. In the example of eliciting your subjective probability of event  $A$  you were offered to choose between a (1) lottery ticket  $P(\text{win}) = n/100$  and a (2) possible reward of same value if event  $A$  occurs. What is the expected value  $E_i$  of your gain if you choose option  $i$ ? ( $i \in \{1, 2\}$ ). Solve  $P(A)$  from the equation  $E_1 = E_2$ .
3. Grandfather goes shopping in four shops. In each shop, he can forget his umbrella with probability 0.25, and remembers it with probability 0.75. After he has gone through all four shops, what is the probability that the umbrella was forgotten in the third shop? What is the probability that it was forgotten in the third shop, given that he really did forget it in one of the shops?
4. Assume the goal is to infer about some quantity  $X$  based on observations about another quantity  $Y$ . If the joint distribution  $P(X, Y)$  is known, either  $P(Y | X)$  or  $P(X | Y)$  could be derived from this. If  $X$  is informative about  $Y$  (i.e.  $P(Y | X) \neq P(Y)$ ), or vice versa, does it mean that there must be a causal relationship between the two? Find supporting examples.
5. Continue Exercise 1.2. Assume there is some genotype in humans that can protect from serious infections, and the population prevalence of this type is  $g = 0.03$ . If a person has the genotype and is infected, the conditional probability of survival is one, regardless of the virus type. If a person does not have the genotype, assume the conditional probabilities only depend on the influenza type as given in the exercise. Assume a patient survived infection that was one of the types  $T_1, T_2, T_3$ . What is the probability that he had this advantageous genotype?
6. Approximately  $1/125$  of all births are fraternal twins and  $1/300$  identical twins. Elvis Presley had a twin brother who died at birth. What is the probability that Elvis was an identical twin?
7. Posterior of  $r$  was calculated in two steps: after observing one red ball, and then using this posterior as a new prior when the 2nd ball was observed red. Verify that we get the same posterior probability formula by using  $P(r = i/N) = 1/(N + 1)$  as the prior, and  $P(1^{\text{st}} X = \text{red}, 2^{\text{nd}} X = \text{red} | r = i/N)$  as the model for both observations.
8. You strongly suspect your car must have either electricity problems, or some other problems, but not both. If the car has electricity problems, the key's remote control button won't open the door with probability  $P(\text{not open} | \text{e-problem}) = 0.1$ . Otherwise, the probability is  $P(\text{not open} | \text{other problem}) = 0.2$ . Then,  $P(\text{e-problem}) + P(\text{other problem}) = 1$ , and  $P(\text{no problem}) = 0$ . Now, you push the key-button and nothing happens. What is the probability that the car really has e-problems? Discuss how this depends on your initial beliefs about the existence of e-problems versus other possible problems. (Plot the posterior as a function of prior). In this model, have you included the possibility that you might have wrong keys?
9. When dad bakes gingerbread, they get burned with probability 0.5. When mom bakes, they get burned with probability 0.9. Gingerbread is made every month. Every other month dad is cooking,

and mom is cooking for other months. One day, their son was served with unburned gingerbread. What is the probability, that dad was the cook? What if we know dad is cooking for one month per year only? What prior probability would give  $<10\%$  posterior probability?