1.1 Exercises

1. Microbiological sampling is repeated three times under same conditions. Assume the probability to catch salmonella in each is p. What is the probability $P(A)$ for the event $A =$"at least one of the three samples contains salmonella"? Assume that the sample of three is not analyzed as three separate tests but as a single pooled sample, and the detection probability of the pooled sample is q given that at least one of the three subsamples contained salmonella. What is the probability for the event $B =$"negative result for the pooled test", and $\bar{B} =$"positive result for the pooled test". Show that they add up to one.

2. Assume that influenza types T_1, T_2 and T_3 each will spread into a population, reaching prevalence $p_{T_1} = 0.05, p_{T_2} = 0.1$ and $p_{T_3} = 0.3$. Assume that influenza specific mortalities are $P(\dagger \mid T_1) = 0.005$, $P(\dagger \mid T_2) = 0.0001$, and $P(\dagger \mid T_3) = 0.00001$. For simplicity, assume also that an individual can only acquire one of the diseases or none. What is the probability of death ($P(\dagger)$) for an arbitrary citizen?

3. To continue the previous exercise, assume that a disease of type T_i will cause costs C_i per each case of illness. The expected values of the costs are $E(C_1) =$ EUR 400, $E(C_2) =$ EUR 100, $E(C_3) =$ EUR 50. What is the expected cost for an arbitrary citizen? Use the conditional expected value in calculations. (Hint: indicator variable may help).

4. Assume that the joint probability for X (rain='yes'/'no'=1/0) and Y (windy='yes'/'no'=1/0) is given in the table below. (Chosen numbers are purely fictional).

	Y=0	Y=1
X=0	0.3	0.4
X=1	0.1	0.2

What is the conditional probability that it rains if it is windy? What is the conditional probability that it is windy if it rains? What is the marginal probability of rain, and the marginal probability of wind? Are rain and wind independent?

5. Extend the previous model of X, Y with a new variable Z (cloudiness='High'/'Low') so that X and Y have the following conditional probabilities (again fictional), given Z:

	Z=0		Z=1	
	Y=0	Y=1	Y=0	Y=1
X=0	0.45	0.45	0.15	0.35
X=1	0.05	0.05	0.15	0.35

Are X and Y conditionally independent, given Z? Verify that with $P(Z = 1) = 0.5$, this extended model gives the previous table of marginal probabilities $P(X, Y)$.

6. If X and Y are independent, show that $P(X \mid Y) = P(X)$.

7. Using product rule, factorize the joint density $\pi(x, y, z)$ into a product of three parts.

8. Broiler flocks can be either infected or not infected with probability p. Define an indicator variable for each flock $i = 1, \ldots, n$:

$$I_i = \begin{cases} 1 & \text{if flock } i \text{ is infected} \\ 0 & \text{otherwise.} \end{cases}$$

Assume that the flock size has some distribution with mean μ and the flocks are independent. Show that the expected number of broilers in infected flocks is $n\mu p$.

9. Show that the function $\pi(x) = -\ln(x)$ is a probability density over the interval $(0, 1]$. (You need this detail: $\lim_{x\to 0^+} x\ln(x) = 0$).

10. X has uniform probability density over $(0, 1]$, ($\pi(x) = 1$ if $x \in (0, 1]$ and zero elsewhere). What is the probability density of $Y = \ln(X)$? What is its support? Show that it integrates to one.