Elements of Set Theory, Fall 2009

Exercise 7

13.11.2009

1. Show that if x and y are real numbers with $x <_{\mathbb{R}} y$, then there is a rational number r such that $x <_{\mathbb{R}} r_{\mathbb{R}} <_{\mathbb{R}} y$.
2. Show that the union of two finite sets is finite (directly, without using Theorem 6J of Enderton's.)
3. Show that the equation $f(m,n) = 2^m(2n+1)-1$ defines a one-to-one function from $\omega \times \omega$ onto ω.
4. Let K, L and M be pairwise disjoint sets. Prove:
   - $K \times (L \cup M) \approx (K \times L) \cup (K \times M)$,
   - ${}^M(K \times L) \approx {}^M K \times {}^M L$.
5. Suppose that κ is a nonzero cardinal number. Show that there is no set to which every set of cardinality κ belongs.
6. Here $+$ and $\cdot$ refer to the addition and multiplication of cardinal numbers, respectively. Recall that $\aleph_0 = |\omega|$. Prove:
   (a) $n + \aleph_0 = \aleph_0$ for every $n \in \omega$,
   (b) $n \cdot \aleph_0 = \aleph_0$ for every $n \in \omega$,
   (c) $\aleph_0 + \aleph_0 = \aleph_0$,
   (d) $\aleph_0 \cdot \aleph_0 = \aleph_0$.