# Elements of Set Theory, Fall 2009

Exercise 5

23.10.2009

1. Show that even though for given sets $A$ and $B$, the collection of functions from $A$ to $B$ is a set, the collection of *all* functions is not a set.

2. Is there a function $F : \mathbb{Z} \to \mathbb{Z}$ satisfying the equation

$$F([\langle m, n\rangle]) = [\langle m + n, n\rangle]?$$

3. Suppose that $R$ is a binary relation on a set $A$, and $\sim$ is an equivalence relation on $A$. Define that $R$ is *compatible* with $\sim$ if whenever $x \sim x'$, $y \sim y'$, and $xRy$, also $x'Ry'$. Prove that there exists a relation $\bar{R}$ on $(A/\sim) \times (A/\sim)$ satisfying

$$[x]\bar{R}[y] \Leftrightarrow xRy$$

   if and only if $R$ is compatible with $\sim$.

   *Note:* If $A$ has at least 2 elements, we can infer this result from the analogous result for functions, as we did in the lectures.

4. Give an example of a set $A$, an equivalence relation $\sim$ on $A$, and a binary relation $R$ on $A$ such that $R$ is not compatible with $\sim$. Explicate why there is no relation $\bar{R}$ as in the exercise above. *Suggestion:* Let for example $A = \{0, 1\}$, $\sim = A \times A$, and $R = \{\langle 0, 1\rangle\}$.

5. Verify that the multiplication of rationals is associative and commutative:

$$(p \cdot_{\mathbb{Q}} q) \cdot_{\mathbb{Q}} r = p \cdot_{\mathbb{Q}} (q \cdot_{\mathbb{Q}} r),$$

$$p \cdot_{\mathbb{Q}} q = q \cdot_{\mathbb{Q}} p$$

   for every $p, q, r \in \mathbb{Q}$.

6. Give a direct proof (not using Theorem 5QF of Enderton's) that if $r$ and $s$ are nonzero rational numbers, then $r \cdot_{\mathbb{Q}} s$ is also nonzero.