Elements of Set Theory, Fall 2009

Exercise 1

25.9.2009

As agreed in the class, you will be awarded the points of one problem (6pt) in the exam if you solve 50% of the exercises during the semester, and you will be awarded 9pt if you solve 75%. (*Solving* means here that you have honestly tried to give a solution to the problem.)

1. (a) Show that if $A \subseteq B$, then $\bigcup A \subseteq \bigcup B$ and $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.
   (b) Show that for any set A, $\bigcup \mathcal{P}(A) = A$.
   (c) Assume that $\mathcal{P}(A) = \mathcal{P}(B)$. Show that $A = B$.

2. Prove *De Morgan's Laws*:

$$C \setminus (A \cup B) = (C \setminus A) \cap (C \setminus B)$$

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3. Show that there is no set to which every singleton set belongs. (Hint: if A was such a set, construct a universal set from A.)

4. Suppose that A is a finite set with n elements. Show that the number of elements in $\mathcal{P}(A)$ is 2^n. (Hint: induction on n.)

5. Recall that we defined $\operatorname{dom}(A)$ and $\operatorname{ran}(A)$ for *any* set A. Show that A is a relation if and only if $A \subseteq \operatorname{dom}(A) \times \operatorname{ran}(A)$.

6. Suppose that we attempted to generalize the Kuratowski definition for ordered pairs to ordered triples by defining

$$\langle x, y, z \rangle^* = \{\{x\}, \{x, y\}, \{x, y, z\}\}.$$

   Show that this definition is unsuccesful by giving examples of sets x, y, z, u, v, w such that $\langle x, y, z \rangle^* = \langle u, v, w \rangle^*$ but $y \neq v$ or $z \neq w$ (or both).