Lin models and BUGS

Normal model for data y_i, i=1,…,n:

- $y_i \sim N(\mu_i,\tau)$ with $\tau=1/\sigma^2$

  and $\mu_i = \alpha_0 + \alpha_1 X_{i1} + \ldots + \alpha_p X_{ip}$

- α_0 is **intercept** (mean of y when all X are at zero values).
- α_j is **slope** for covariate X_j. When X_j is increased by one unit, the mean μ_i is increased by α_j.
- Could predict y_i for some hypothetical X_i*

Lin models and BUGS

- Conditional joint probability density of all data is:

$$\pi(y|\alpha,\tau) = \prod_{i=1}^{n} \mathrm{N}(y_i|\mu_i,\tau)$$

(conditional independence, given μ_i , τ)

- This is also "likelihood function" of α,τ:

  $L(\alpha,\tau ; y)$ for given data y.

Lin models and BUGS

- Posterior density of parameters:

$$\pi(\alpha,\tau|y) \propto \prod_{i=1}^{n} \mathrm{N}(y_i|\mu_i,\tau) \times \pi(\alpha,\tau)$$

- in BUGS:

```
for(i in 1:n){
y[i] ~ dnorm(mu[i],tau); mu[i] <- inprod(a[],X[i,]) }
tau ~ dgamma(0.001,0.001)
for(j in 1:p){ a[j] ~ dnorm(0,0.001) }
```

Lin models and BUGS

- Easy to experiment with different priors in BUGS, also multivariate priors (e.g. Zellner's)

  → how sensitive are the results?

- May be useful to standardize covariates, to reduce posterior correlation between parameters, and possibly for better MCMC convergence.

ANOVA and BUGS

- Covariates **X are categorical variables**.
- Measurement y is obtained from some individuals (experiments) within some category, defined by X.

- Using the intercept term, the mean in category X_j would be $\alpha_0 + \alpha_j$ for all measurements there.

  → parameters α_0 and α_j not identifiable from data as such.

  → Need constraints: CR or STZ,

  or use ”simple model” without α_0.

ANOVA and BUGS

- Only one categorical covariate: ”one way anova”
- Two categorical covariates: ”two way anova”
  - only main effects included, or
  - interactions included (then all the corresponding main effects need to be included)
- Compare the main effect parameters to see if different levels of categorical covariates have an effect on response. (e.g. box plots in BUGS).
  - e.g. means at different age groups → is there a pattern? (linearity over groups is not assumed).

ANOVA and BUGS

- Estimate interactions to see if some combination of categorical covariate levels in A and B have a combined effect that could not be explained by their main effects only.
  - i.e. the mean response in category $A_k B_r$ may not be simply described by $\alpha_k + \beta_r$ but as $\alpha_k + \beta_r + \gamma_{k,r}$.
  - interaction terms would again lead to unidentifiability → need CR or STZ for them too.
  - Multifactor analysis, higher order interactions, but try to keep it as simple as possible.

ANOVA and BUGS

- Typical covariates:
  - Age groups
  - Gender
  - ’Treatment’ groups
    - drug/placebo
    - smoking/non-smoking
    - dose groups: low dose, medium dose, high dose.
  - Other classification, e.g. econ status, residence.
- Aim to estimate (main) effects of these, and possible interaction effects.

ANOVA and BUGS

- BUGS:
  - **individual data format**:

    for(i in 1:n){ y[i] ~ dnorm(mu[i],tau) }

    where we loop over n individual measurements.
  - **tabular data format**:

    for(k in 1:K){

    for(r in 1:R){ y[k,r] ~ dnorm(mu[k,r],tau) }}

    where we loop over R measurements in K categories.
  - define mu[i] or mu[k,r] accordingly !

ANOVA and BUGS

- BUGS, individual data format:
  - Could write out the linear predictor as

    mu[i] <- a0 + a1*equals(X[i],1) + … + ap*equals(X[i],p)

    for different levels of categorical variable X.
  - Need CR or STZ for a0,…,ap, **or** eliminate a0.
  - Could use **dummy variables**, which are defined according to CR or STZ, so that the constraints become imposed via these dummies.
  - Compact expression: inprod(a[],X[i,])
  - Similar idea for tabular data format.

ANCOVA and BUGS

- In the same model, in linear predictor, we include:
  - Continuous covariates X, as before with normal linear models.
  - Categorical covariates X, as before with anova models
  - Could model e.g. different regression lines in each categorical group.
    - special case: common slope across groups
    - special case: common intercept across groups
    - etc
  - BUGS implementation is analogous to previous.

GLM and BUGS

- Extending the concept of linear models.
- E.g. Binomial data $y_i \sim Bin(p_i, n_i)$, where we want to explain p by some covariates X. Link function needed:

$$logit(p_i) = \alpha_0 + \alpha_1 X_{i1} + \ldots + \alpha_p X_{ip}$$

When all X=0, we get $p_0 = \exp(\alpha_0)/(1+\exp(\alpha_0))$

and if also $\alpha_0 = 0$, then $p_0=0.5$.

p_0 could represent the reference category, if X are categorical, or the success probability for an individual with all X=0, if X is continuous.

GLM and BUGS

- Typically, noninformative priors as default.
- BUGS:

```
for(i in 1:n){
y[i] ~ dbin(p[i],n[i]); logit(p[i]) <- inprod(a[],X[i,]) }
 for(j in 1:p){ a[j] ~ dnorm(0,0.001) }
```

- Make use of available link functions as above, or solve p[i] = f(inprod(a[],X[i,])) and write it as expression.

GLM and BUGS

- Different link functions in different GLMs, e.g. log-linear link in Poisson models.
- Interpretation of GLM parameters depends on the link.
  - e.g. with log-linear Poisson model:

    $Log(\lambda_i) = \alpha_0 + \alpha_1 X_{i1} + \ldots + \alpha_p X_{ip}$

    the effect of increasing covariate X_j by one unit is multiplicative for λ_i: $\exp(\alpha_j)$ → this could be monitored in BUGS for reporting the results. A positive effect $\alpha_j > 0$ corresponds to $\exp(\alpha_j) > 1$.

LM, ANOVA, ANCOVA, GLM and BUGS

- In all the above, practical interest could be
  - about **estimates** of model parameters in the linear predictor. These describe effects of covariates of interest. In $N(\mu_i,\sigma^2)$-models we also get estimates of σ, which may not be of primary interest (but uncertainty about it will be counted in the joint posterior distribution). Might study $1-\sigma^2/s^2$ to see the proportional reduction of uncertainty we achieve by incorporating the X in our model.
  - about **predictions** of y, under different X.

LM, ANOVA, ANCOVA, GLM and BUGS

- DIC readily available for model comparison, but not always applicable! (See warnings in manual).
- Missing data in y will be predicted. (No effect on parameter estimates).
- Missing data in x will reguire a specification of π(x | parameters). If the corresponding y is known, this will influence parameter estimates, because it will describe what are probable values for the missing x. Technically: just add the conditional model for x in BUGS code.

LM, ANOVA, ANCOVA, GLM and BUGS

- Censored data y:
  - left censoring
  - right censoring
  - interval censoring
  - implement using I(,)-function which will add the correct ”likelihood contribution” $P(y_i > a)$, $P(y_i < b)$, or $P(a < y_i < b)$ from each such observation.
  - truncated distribution is a different problem. Will be better addressed in OpenBUGS (T(,)-function).
- Distributions not directly available in BUGS:

  Try zeros trick or ones trick.