Intro2 in nutshell

Using posterior distribution $\pi(\theta|data)$:

- Summarizes current uncertainty about unknown quantities θ, given observed (hence fixed) data.
- Can be approximated by histogram of simulated sample $\theta_1,\ldots,\theta_n$.
- Likewise, posterior mean is approximated as the average $\Sigma\theta_i/n$.
- Posterior mean of any function $g(\theta)$ is approximated as $\Sigma g(\theta_i)/n$.

Intro2 in nutshell

• Can be used to compute **$P(H_0 \mid data)$**, when H_0 is expressed as a function of θ. From simulated posterior distribution:

$$P(H_0 \mid data) \approx (\#\ \text{iterations}\ H_0(\theta_i)\ \text{is true})/n.$$

• Bayesian **Credible Intervals** (CI) for summaries of the uncertainty width. From simulated posterior: sort the sample $\theta_1,...,\theta_n$, take 2.5% of the smallest values, 2.5% of the largest values out, to get 95% credible interval. For HPD interval, you need to sort with respect to posterior density values $\pi(\theta_1|data),...,\pi(\theta_n|data)$ and take out 5% of the smallest.

• **Posterior predictive distributions**. From simulated posterior, generate predicted observation X_i from $\pi(X_i \mid \theta_i)$ for each θ_i.

Intro2 in nutshell

- Monte Carlo simulations: iid samples from the target density.
  - –Efficient if every sample is drawn from the target density.
  - –Rejection samplers: iid sampling, but only some draws are accepted.

- MCMC simulations: dependent samples from the target density. Need to check convergence.
  - –If full conditionals can be solved → Gibbs, every draw is accepted in ’alternating sampling’.
  - –More generally: MH-algorithm with proposal density, but acceptance rate should not be too high, nor too low.
  - –Single site updating vs. block updating.