

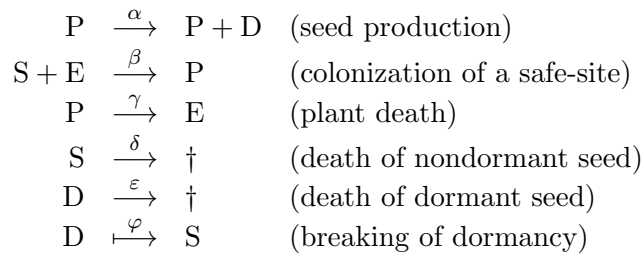
STOCHASTIC POPULATION MODELS

EXERCISES 6

6.

The previous exercise 5 led to a system of DDEs. This was not what I intended. I expected to get a single DDE to which we could apply the stability analysis as we developed in the lectures. Below I start with the same processes that previously (i.e., in section 1.9 of the lecture notes) led to the logistic equation, but now with a delay not in plant development (as in exercise 5) but in seed development. I will give the final equation, but it might be good if you tried to derive it yourself.

Let P denote an individual plant, D a dormant (=inactive) seed, S a nondormant (=active) seed, E an empty safe site, and consider the following processes:



Assuming that seed production and death of nondormant seeds are fast compared to the other processes, we arrive at

$$\frac{dp}{dt} = \frac{\alpha\beta(e_0 - p)}{\delta} \int_0^\infty p(t - \tau)e^{-\varepsilon\tau}\varphi(\tau)d\tau - \gamma p$$

Assuming a fixed length τ of the dormancy period, this becomes

$$\frac{dp}{dt} = \frac{\alpha\beta e^{-\varepsilon\tau}}{\delta} p_\tau(e_0 - p) - \gamma p$$

where $p_\tau(t) = p(t - \tau)$ for all t .

- (a) If you do not understand where the above DDEs come from, do the derivation yourself.
- (b) Calculate the equilibria of the latter DDE. Under what conditions is there a positive equilibrium?
- (c) Determine the local stability of the positive equilibrium whenever it exists. Use the approach used in section 3.3 of the lecture notes.