STOCHASTIC POPULATION MODELS

EXERCISES 6

6.
The previous exercise 5 led to a system of DDEs. This was not what I intended. I expected to get a single DDE to which we could apply the stability analysis as we developed in the lectures. Below I start with the same processes that previously (i.e., in section 1.9 of the lecture nots) led to the logistic equation, but now with a delay not in plant development (as in exercise 5) but in seed development. I will give the final equation, but it might be good if you tried to derive it yourself.

Let P denote an individual plant, D a dormant (=inactive) seed, S a nondormant (=active) seed, E an empty safe site, and consider the following processes:

$$\begin{array}{rcll} \mathrm{P} & \xrightarrow{\alpha} & \mathrm{P}+\mathrm{D} & \text{(seed production)} \\ \mathrm{S}+\mathrm{E} & \xrightarrow{\beta} & \mathrm{P} & \text{(colonization of a safe-site)} \\ \mathrm{P} & \xrightarrow{\gamma} & \mathrm{E} & \text{(plant death)} \\ \mathrm{S} & \xrightarrow{\delta} & \dagger & \text{(death of nondormant seed)} \\ \mathrm{D} & \xrightarrow{\varepsilon} & \dagger & \text{(death of dormant seed)} \\ \mathrm{D} & \xmapsto{\varphi} & \mathrm{S} & \text{(breaking of dormancy)} \end{array}$$

Assuming that seed production and death of nondormant seeds are fast compared to the other processes, we arrive at

$$\frac{dp}{dt} = \frac{\alpha\beta(e_0 - p)}{\delta} \int_0^\infty p(t-\tau)e^{-\varepsilon\tau}\varphi(\tau)d\tau - \gamma p$$

Assuming a fixed length τ of the dormancy period, this becomes

$$\frac{dp}{dt} = \frac{\alpha\beta e^{-\varepsilon\tau}}{\delta} p_\tau(e_0 - p) - \gamma p$$

where $p_\tau(t) = p(t-\tau)$ for all t.

(a) If you do not understand where the above DDEs come from, do the derivation yourself.

(b) Calculate the equilibria of the latter DDE. Under what conditions is there a positive equilibrium?

(c) Determine the local stability of the positive equilibrium whenever it exists. Use the approach used in section 3.3 of the lecture notes.