SECOND COURSE IN STATISTICS 20.1.–28.4.2011. Literature: Sheldon M. Ross: Introductory Statistics. Lecturer: University Lecturer Pekka Pere.

1st set of exercises 27.1.2011

1. Let X be a random variate and $\mathsf{E}(X) = \mu$. Explain the conceptual difference, if any, between the expected value (of X), the population mean, the sample mean, and the sample average.

2. Let us assume that X_i is a random variate with mean μ_i and that c and a_i are constants ($i = 1, \ldots, k$). It then holds that (pp. 221–222):

$$\begin{aligned} \mathsf{E}(cX_i) &= c\mathsf{E}(X_i) = c\mu_i, \\ \mathsf{E}(c + X_i) &= c + \mathsf{E}(X_i) = c + \mu_i, \\ \mathsf{E}(X_i + X_j) &= \mathsf{E}(X_i) + \mathsf{E}(X_j) = \mu_i + \mu_j. \end{aligned}$$

Use these results to prove that

$$\mathsf{E}\left[c + \sum_{i=1}^{k} a_i X_i\right] = c + \sum_{i=1}^{k} a_i \mu_i.$$

3. Let us assume that X_is are independent random variates with variance σ_i^2 ($i = 1, \ldots, k$). It then holds that (pp. 233–234):

$$\begin{aligned} \mathsf{var}(cX_i) &= c^2\mathsf{var}(X_i)) = c^2\sigma_i^2, \\ \mathsf{var}(c + X_i) &= \mathsf{var}(X_i) = \sigma_i^2, \\ \mathsf{var}(\textstyle\sum_{i=1}^{k} X_i) &= \textstyle\sum_{i=1}^{k} \mathsf{var}(X_i) = \sum_{i=1}^{k} \sigma_i^2. \end{aligned}$$

Use these results to prove that

$$\mathsf{var}\left[c + \sum_{i=1}^{k} a_i X_i\right] = \sum_{i=1}^{k} a_i^2 \sigma_i^2.$$

4. Let X_1 and X_2 be two random variables with means μ_1 and μ_2 and variances σ_1^2 and σ_2^2, respectively.

a) Let us assume that the variables are independent. What is the variance of $X_1 + X_2$? What is the variance of $X_1 - X_2$?

b) Let us give up the independence assumption and assume that $X_1 = X_2$. What is the variance of $X_1 + X_2$? What is the variance of $X_1 - X_2$?

5. The market price of a stock is 100 euros (at the OMX Helsinki exchange, say).

a) What is the expected value of the stock if the price of it will rise by 5 % with probability $0,7$ and will fall by 25 % with probability $0,3$?

b) Let us assume that an investor holds the stock if its expected value is higher than (or the same as) the present market price and sells the stock if the expected value of it is smaller than the present market price. Is it possible that the probability that the price of the stock raises is higher than the probability that it falls, and nevertheless the investor sells his stock? Explain carefully.

c) A recent Finnish book for investors reads as follows[1]:

Expected return is a theoretical return based on a long run of statistics, a figure in the middle of the density function of the returns. Usually the density is depicted as a bell shaped curve also known as the Gaussian curve [the normal density function] *in which the expected return is the point at the top of the density.*

What is wrong in this explanation? Explain.

[1] The translation is mine. I have added a clarification in the square brackets. The original text is: *Odotettu tuotto on pitkän ajan tilastoihin perustuva, keskimääräinen teoreettinen tuotto, keskiluku tuottojen todennäköisyysjakaumassa. Tavallisesti jakauma esitetään niin sanottuna kellokäyränä eli Gaussin käyränä* [normaalijakauma]*, jossa odotettu tuotto on siis käyrän huippupiste.*