Homework exercises, set 2

1. Show that the wedge product $f \wedge g$ of $f \in \Omega^k(V)$ and $g \in \Omega^l(V)$ is really in $\Omega^{k+l}(V)$.

2. With the notation of (1), show that $f \wedge g = (-1)^{kl} g \wedge f$.

3. Prove that the wedge product is associative.

4. Let $X, Y \in \mathcal{D}^1(M)$ and $\omega \in \Omega(M)$. Show that $\mathcal{L}_X(\mathcal{L}_Y \omega) - \mathcal{L}_Y(\mathcal{L}_X \omega) = \mathcal{L}_{[X,Y]}\omega$. Hint: Do first the case $\omega \in \Omega^1(M)$ and then generalize to forms of arbitrary degree.

5. Prove the relation $\mathcal{L}_X = d \circ i_X + i_X \circ d$.

6. Use the result of exercise 5 to show that Lie derivative and exterior differentiation commute.