

Logiikan pääs
 Ehto-kaikisuu ehdotuksia 3

$$1. \frac{\frac{B \wedge A}{A} \quad \frac{B \wedge A}{B}}{A \vee B} \vee I$$

$$2. \frac{\frac{\{A\} \vee I}{A \vee B} \quad \frac{\{B\} \vee I}{A \vee B}}{A \vee B} \vee E$$

$$3. \frac{\frac{A}{A \vee A} \vee I}{A \vee (A \vee A)} \vee I$$

$$4. \frac{A \vee (A \vee A) \quad \frac{\{A\} \vee I \quad \frac{\{A\} \vee I \quad \{A\} \vee I}{A} \vee E_1}{A} \vee E_2$$

$$5. \frac{\frac{A \wedge C}{A} \wedge E \quad \frac{A \wedge C}{C} \wedge E}{A \wedge (B \vee C)} \wedge I$$

$$6. \frac{\frac{\frac{A \wedge B}{B} \wedge E \quad \frac{A \wedge B}{A} \wedge E}{B \wedge (A \vee C)} \wedge I \quad \frac{\frac{B \wedge C}{B} \wedge E \quad \frac{B \wedge C}{C} \wedge E}{B \wedge (A \vee C)} \wedge I}{B \wedge (A \vee C)} \vee E$$

$$7. \frac{\frac{\frac{(A \vee B) \wedge (B \vee C)}{A \vee B} \wedge E \quad \frac{\{B\} \vee I}{B \vee (A \wedge C)} \vee I}{B \vee (A \wedge C)} \vee E \quad \frac{\frac{(A \vee B) \wedge (B \vee C)}{B \vee C} \wedge E \quad \frac{\{B\} \vee I}{B \vee (A \wedge C)} \vee I}{B \vee (A \wedge C)} \vee E}{B \vee (A \wedge C)} \vee E_2$$

$$8. \frac{A \rightarrow (B \rightarrow C) \quad \{A\} \vee I}{A \rightarrow C} \rightarrow E$$

$$\frac{B \rightarrow C \quad \{B\} \vee I}{C} \rightarrow I_1$$

$$\frac{A \rightarrow C}{B \rightarrow (A \rightarrow C)} \rightarrow I_2$$

9.

$$\begin{array}{c}
 \frac{\frac{[A \wedge B]^1}{A} \wedge E \quad \frac{[A \wedge B]^1}{B} \wedge E}{A \wedge B} \wedge I \\
 \frac{(B \wedge A) \rightarrow (D \wedge C) \quad B \wedge A}{(B \wedge A) \rightarrow (D \wedge C)} \rightarrow E \\
 \frac{D \wedge C}{C} \wedge E \\
 \frac{C \quad D}{C \wedge D} \wedge I \\
 \frac{C \wedge D}{(A \wedge B) \rightarrow (C \wedge D)} \rightarrow I
 \end{array}$$

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$$\begin{array}{c}
 \frac{[A \rightarrow B]^1 \quad [A]^2}{A \rightarrow B} \rightarrow E \quad \frac{[A \rightarrow C]^1 \quad [A]^2}{A \rightarrow C} \rightarrow E \\
 \frac{B}{B \vee C} \vee I \quad \frac{C}{B \vee C} \vee I \\
 \frac{B \vee C}{A \rightarrow (B \vee C)} \rightarrow I_2 \\
 \frac{A \rightarrow (B \vee C)}{(A \rightarrow B) \vee (A \rightarrow C)} \vee E_1
 \end{array}$$

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$$\begin{array}{c}
 \frac{[A \rightarrow B]^3 \quad [A]^1}{A \rightarrow B} \rightarrow E \\
 \frac{B}{\neg B} \wedge I \\
 \frac{B \wedge \neg B}{\neg A} \wedge I_1 \\
 \frac{\neg A}{\neg B \rightarrow \neg A} \rightarrow I_2 \\
 \frac{\neg B \rightarrow \neg A}{(A \rightarrow B) \rightarrow (\neg B \rightarrow \neg A)} \rightarrow I_3
 \end{array}$$

12.

$$\begin{array}{c}
 \frac{[\neg B \rightarrow \neg A]^3 \quad [\neg B]^1}{\neg A} \rightarrow E \\
 \frac{\neg A}{A \wedge \neg A} \wedge I \\
 \frac{A \wedge \neg A}{\neg B} \wedge I_1 \\
 \frac{\neg B}{A \rightarrow B} \rightarrow I_2 \\
 \frac{A \rightarrow B}{(\neg B \rightarrow \neg A) \rightarrow (A \rightarrow B)} \rightarrow I_3
 \end{array}$$

13.

$$\frac{\frac{\frac{[A]^1}{AV\bar{A}} \vee I}{(\bar{A} \vee \bar{A})} \wedge I}{(\bar{A} \vee \bar{A}) \wedge \bar{A}} \wedge I$$

$$\frac{\frac{\bar{A}}{AV\bar{A}} \vee I}{(\bar{A} \vee \bar{A}) \wedge \bar{A}} \wedge I$$

$$\frac{\frac{[A]^1}{AV\bar{A}} \vee I}{(\bar{A} \vee \bar{A}) \wedge \bar{A}} \wedge I$$

$$\frac{\bar{A} \vee \bar{A}}{\bar{A} \vee \bar{A}} \wedge I$$

$$\frac{\bar{A} \vee \bar{A}}{AV\bar{A}} \wedge I$$

14.

$$\frac{\frac{[A]^2}{B \rightarrow A} \rightarrow I}{(B \rightarrow A) \vee (A \rightarrow B)} \vee I$$

$$\frac{AV\bar{A}}{(B \rightarrow A) \vee (A \rightarrow B)} \vee I$$

$$\frac{\frac{[A]^1}{A \wedge \bar{A}} \wedge I}{\bar{A}} \wedge I$$

$$\frac{\bar{A}}{A \rightarrow B} \rightarrow I$$

$$\frac{A \rightarrow B}{(B \rightarrow A) \vee (A \rightarrow B)} \vee I$$

$$\frac{(B \rightarrow A) \vee (A \rightarrow B)}{(B \rightarrow A) \vee (A \rightarrow B)} \vee I$$

15. Yhdistämällä tehtävien 13 ja 14 tulokset saadaan haluttu päätelmä.

Tutkitaan lausekka $((A \vee B) \rightarrow C) \leftrightarrow ((A \rightarrow C) \wedge (B \rightarrow C))$

16.

A	B	C	$((A \vee B) \rightarrow C)$	\leftrightarrow	$((A \rightarrow C) \wedge (B \rightarrow C))$
1	1	1	1	1	1
1	1	0	0	0	0
1	0	1	1	1	1
1	0	0	0	0	0
0	1	1	1	1	1
0	1	0	0	0	0
0	0	1	1	1	1
0	0	0	1	0	0

lause on tautologia, joten $(A \vee B) \rightarrow C$ ja $(A \rightarrow C) \wedge (B \rightarrow C)$ loog. ekv.

17.

$(p_0 \rightarrow \neg p_1)$	\wedge	\neg	$(p_1 \rightarrow \neg p_0)$
1 0 0 1	0	1	1 0 0 1
1 1 1 0	0	0	0 1 0 1
0 1 0 1	0	0	1 1 1 0
0 1 1 0	0	0	0 1 1 0

ristriittä \uparrow

DNF muodossa: $p_0 \wedge \neg p_0$

18.

$(p_0 \wedge p_1)$	\rightarrow	$(p_2 \wedge p_1)$
1 1 1	1	1 1 1
1 1 1	0	0 0 1
1 0 0	1	1 0 0
1 0 0	1	0 0 0
0 0 1	1	1 1 1
0 0 1	1	0 0 1
0 0 0	1	1 0 0
0 0 0	1	0 0 0

DNF muodossa: $\neg p_0 \vee \neg p_1 \vee p_2$

19.

jos $v(p_0) = 0$ niin $v(p_0 \rightarrow (p_0 \wedge p_1)) = 1$

jos $v(p_0) = 1$ ja $v(p_1) = 0$ niin $v(p_0 \wedge (p_0 \wedge p_1)) = 0$

joten lause on kontingentti.