

Inverse Problems, Problems session 2

1. Suppose $f, h \in \mathcal{S}(\mathbb{R}^n)$, $a > 0$ and $b \in \mathbb{R}^n$. Show that

a) $(\widehat{f * h})(\xi) = \hat{f}(\xi)\hat{h}(\xi)$,

b) $\int_{\mathbb{R}^n} f(z)\hat{h}(z)dz = \int_{\mathbb{R}^n} \hat{f}(z)h(z)dz$,

c) $\mathcal{F}_{x \rightarrow \xi}(f(ax)) = a^{-n}\hat{f}\left(\frac{\xi}{a}\right)$, $\mathcal{F}_{x \rightarrow \xi}(e^{ib \cdot x}f(x)) = \hat{f}(\xi - b)$.

2. Go through the proof of the result

$$\hat{\phi}(z) = c\phi(z), \quad \phi(x) := \exp(-|x|^2/2), \quad z \in \mathbb{R}^n,$$

and be prepared to present the proof on the board.