

Inverse Problems, Problems session 1

1. Let $X = L^2(-\pi, \pi)$ and Z be the subspace spanned by orthonormal functions $e_1(x), e_2(x), \dots, e_N(x) \in Z$ and $S_N : X \rightarrow X$ be orthogonal projector on the space Z . Let $f \in X$. Show that the orthogonal projection $S_N f$ is the closest element of Z to f .
2. Compute the Fourier coefficients of the function $f(x) = x$ in $L^2(-\pi, \pi)$.