STOCHASTIC PARTICLE SYSTEMS: EXERCISE 7

1. Consider the setup of exercise 1.2. Now instead of the simple random walk, consider the simple continuous time random walk. I.e., we consider N independent continous time Markov chains whose jump times are exponentially distributed and jumps are distributed according to those of the simple random walk. Find the generator and invariant measure for this process.

Let ν be the Bernoulli-$\frac{1}{2}$ measure on $\mathbb{T}_N^d$ and f a function on $\mathbb{T}_n^d$ so that $\sum_\eta f(\eta)\nu(\eta) = 1$ and $f(\eta) \geq 0$ for all η. Write μ for the measure $f\nu$ (i.e. the measure which is absolutely continous with respect to ν and has Radon-Nikodym derivative f). Define the entropy of μ as $F(\mu) = \int f \log f d\nu = \sum_\eta f(\eta) \log f(\eta)\nu(\eta)$ and the entropy production

$$\sigma(f) = \frac{1}{4} \sum_{x,y \in \mathbb{T}_N^d} E(c(x,y,\eta)(f(\eta^{x,y}) - f(\eta))(\log f(\eta^{x,y}) - \log f(\eta)).$$

2. Show that $0 \leq F(\mu) \leq N^d \log 2$.

3. Show that $F(\mu) = 0$ is equivalent to $f = 1$.

4. Show that $\sigma(f)$ is convex.