

STOCHASTIC PARTICLE SYSTEMS: EXERCISE 7

1. As in the lectures, let $f_x(\eta) = \eta(x) - \rho$ and $g(x) = (f_0, Lf_x)$. Recall that $\hat{g}(k) = \sum_{i,j=1}^n D_{\alpha,\beta} k_\alpha k_\beta + o(k^2)$. Calculate g and $D_{\alpha,\beta}$ explicitly in the case of the simple symmetric exclusion process, i.e. when $p(x, y) = \frac{1}{2d} \mathbf{1}(|x - y| = 1)$.

2. Show that

$$E \left(J_{x,y}[0, t] \int_0^t j(x, y, \eta_s) ds \right) = 0.$$

3. Let $j_\alpha(\eta) = \frac{1}{2} \sum_y y_\alpha j(0, y, \eta)$. Show that

$$\sum_x (S(t, x) - S(0, x)) x_\alpha x_\beta = \frac{t}{2} \sum_x x_\alpha x_\beta E(c(x, 0, \eta)) - 2 \int_0^t ds \int_0^t ds' \sum_x E(j_\alpha(\eta_s) \tau_x j_\beta(\eta_{s'})).$$