STOCHASTIC PARTICLE SYSTEMS: EXERCISE 6

1. Consider a reversible measure μ for a strongly continuous contraction semigroup $S(t)$ on $C(X)$ with generator L. Moreover, let $\tilde{L}$ be the L^2 extension of L considered in the lectures. Show that $\tilde{L}$ is self adjoint.

2. Consider the setup of the previous problem. Recall that in the lectures we showed that

$$-(f, \tilde{L}g) = \frac{1}{2} \sum_{x,y} \int c(x, y, \eta) \overline{(f(\eta^{x,y}) - f(\eta))} (g(\eta^{x,y}) - g(\eta)) \mu(d\eta)$$

for cylinder functions f, g. Show that this holds for $f, g \in \mathcal{D}(\tilde{L})$ and that the series converges absolutely.

3. In addition to the setup of the previous exercises, assume that we are considering the Bernoulli case for the measure μ. Then show that if $g(\eta^{x,y}) = g(\eta)$ for all x, y for μ a.e. η, then g is μ a.e. a constant.

4. Consider a strongly continuous contraction semigroup $S(t)$ with generator L (whose domain is $\mathcal{D}(L)$). Show that for $f \in \mathcal{D}(L^{n+1})$ (e.g. $f \in \mathcal{D}(L^2)$ if $f \in \mathcal{D}(L)$ and $\lim_{t\to 0} t^{-1}(S(t)Lf - Lf)$ exists). Then show that

$$S(t)f = \sum_{k=0}^{n} \frac{t^k}{k!} L^k f + \frac{1}{n!} \int_0^t (t - s)^n S(s) L^{n+1} f ds.$$