STOCHASTIC PARTICLE SYSTEMS: EXERCISE 5

1. During the lectures, we showed that given a strongly continuous contraction semigroup $S(t)$ on a Banach space B and the semigroup's generator L, then B_λ defined on B

$$B_\lambda f = \int_0^\infty e^{-\lambda t} S(t) f dt$$

is the right inverse of the operator $\lambda - L$, i.e., that $(\lambda - L)B_\lambda f = f$ for all $f \in B$. Show that it is also the left inverse (so in fact $B_\lambda = (\lambda - L)^{-1}$).

2. Consider the Banach space $C(X)$, where X is a compact metric space with its measurable structure given by the Borel sets.

a) Suppose L is a Markov pregenerator with domain $\mathcal{D}(L) \subset C(X)$. Show that L has a closure $\bar{L}$ and this is a Markov pregenerator as well.

b) Suppose that L is a closed Markov pregenerator with domain $\mathcal{D}(L) \subset C(X)$. Show that for $\lambda > 0$, the range of $\lambda - L$ is a closed subset of $C(X)$.

3. Let $\rho, \rho' \in (0,1)$ and $\rho \neq \rho'$. Show that in finite volume the Bernoulli measures ν_ρ and $\nu_{\rho'}$ are absolutely continuous with respect to each other, but in infinite volume they are not.

4. a) Show that the following are Markov pregenerators.

i) $L = T - 1$, where T maps non-negative functions of $C(X)$ into non-negative functions of $C(X)$ (X as in problem 2), is defined on all of $C(X)$ and satisfies $T1 = 1$.

ii) Let $X = [0,1]$ and $Lf = \frac{1}{2}f''$ with $\mathcal{D}(L) = \{f \in C(X) : f'' \in C(X), f'(0) = f'(1) = 0\}$.

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b) Show that all of these Markov pregenerators are actually Markov generators. What are the corresponding Markov processes?