5.5. Let's see how to prove the stability of a two-cycle:

Assume first that we would have only one age-class:

$$\underline{N}(0) = \begin{pmatrix} N_1(0) \\ 0 \end{pmatrix}, \quad \underline{N}(1) = \begin{pmatrix} 0 \\ PN_1(0) \end{pmatrix}, \quad \underline{N}(2) = \begin{pmatrix} PN_1(0)\, F(PN_1(0)) \\ 0 \end{pmatrix}$$

We can denote the second iterated map as:

$$N_1(t+2) = PN_1(t)\, F(PN_1(t)) = \Psi(N_1(t)) \quad \text{(with } N_2 = 0 \text{ in even years)}$$

The fixed point is $\hat{N}_1 = \frac{a-1}{bP}$.

Looking at the derivative of the map:

$$\Psi'(\hat{N}_1) = 1 - P\hat{N}_1 \left(-\frac{aP}{(1+b\hat{N}_1 P)^2} \right)$$

$$= 1 - P\hat{N}_1 F(P\hat{N}_1) \frac{bP}{1+bP\hat{N}_1} = 1 - \frac{bP\hat{N}_1}{1+bP\hat{N}_1}$$

$$\Rightarrow \quad 0 < \Psi'(\hat{N}_1) < 1$$

Assume we would now have $\underline{N}(0) = \begin{pmatrix} \hat{N}_1 \\ n \end{pmatrix}$
where $n \ll \hat{N}_1$
and $n \ll P\hat{N}_1$

Then: $\underline{N}(1) = \begin{pmatrix} nF(\hat{N}) \\ P\hat{N}_1 \end{pmatrix}, \quad N(2) = \begin{pmatrix} \hat{N}_1 \\ nPF(\hat{N}_1) \end{pmatrix}$

i.e. the missing year class grew by factor

$$PF(\hat{N}_1) < PF(P\hat{N}_1) = 1 \Rightarrow \text{it can't grow!}$$

(It tries to reproduce in years of high $b(\hat{N}_1 + \hat{N}_2) \rightarrow$ low: $\frac{a}{1+b(\hat{N}_1+\hat{N}_2)}$)

Exercises 5.1 - 5.5

5.1. The projection matrix can be constructed directly from combining the assumptions. Because events are discrete, one can also construct a matrix for each event and apply these in correct order:

1. population growth:

$$\begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix}$$

2. Dispersal & survival:

$$\begin{pmatrix} 1-m & ms \\ ms & 1-m \end{pmatrix}$$

So the population dynamics is given by:

$$N(t+1) = \begin{pmatrix} 1-m & ms \\ ms & 1-m \end{pmatrix} \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix} N(t)$$

$$= \begin{pmatrix} (1-m)\rho_1 & ms\rho_2 \\ \rho_1 ms & (1-m)\rho_2 \end{pmatrix} N(t)$$

5.2. Assume an age-structured population with primitive Leslie-matrix:

$$\begin{pmatrix} F_1 & F_2 & F_3 & \cdots & F_\omega \\ P_1 & 0 & 0 & \cdots & 0 \\ 0 & P_2 & 0 & \cdots & 0 \end{pmatrix}$$

Stable age-distribution is the eigenvector corresponding to the leading eigenvalue:

i.e. $Au = \lambda u \Rightarrow$

$$\lambda u_2 = P_1 u_1$$
$$\lambda u_3 = P_2 u_2$$
$$\lambda u_4 = P_3 u_3$$
$$\vdots$$

and so on!

Choosing $u_1 = \frac{1}{\lambda}$, we get:

$$\Rightarrow \quad u_2 = P_1 \lambda^{-2}$$
$$u_3 = \lambda^{-1} P_2 u_2 = P_2 P_1 \lambda^{-3}$$
$$u_4 = P_1 P_2 P_3 \lambda^{-4} \qquad \text{etc.}$$

Thus: $u_i = \dfrac{l_i}{\lambda^i}$

$\square$

b.) Prove: the reproductive value of age class i can be written as

$$v_i = \frac{1}{\lambda}(F_i v_1 + P_i v_{i+1})$$

Reproductive values are in the left eigenvector:

$$v^T L = \lambda v^T$$

$$\Leftrightarrow \quad v^T = \frac{1}{\lambda} v^T L$$

What is the i'th component of this?

$$v_i = \frac{1}{\lambda}(v_1 \cdot F_i + v_{i+1} \cdot P_i)$$

$$\sum_i \sum_j e_{ij} = \sum_i \sum_j \frac{\partial \lambda}{\partial a_{ij}} \frac{a_{ij}}{\lambda}$$

$$= \sum_i \sum_j v_i u_j \frac{a_{ij}}{\lambda}$$

$$= \sum_i v_i \cdot \sum_j u_j \cdot a_{ij} \left(\frac{1}{\lambda}\right)$$

← This is ith row of vector.

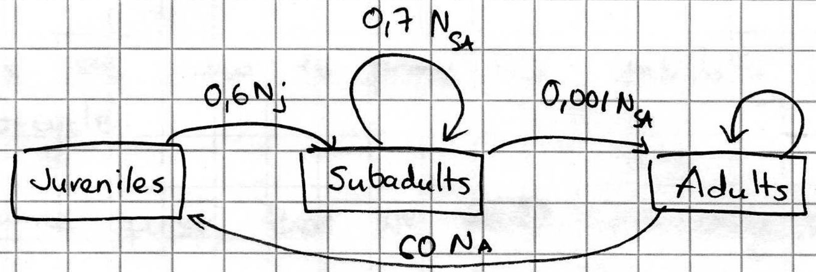
$Au = \lambda u$

because u was eigenvector.

$$= \sum_i v_i \cdot \lambda \cdot u_i \left(\frac{1}{\lambda}\right)$$

$$= \sum_i v_i u_i = \underline{\underline{1}}$$

5,4.



$$\begin{pmatrix} N_j \\ N_{SA} \\ N_A \end{pmatrix} = \begin{pmatrix} 0 & 0 & 60 \\ 0.6 & 0.7 & 0 \\ 0 & 0.001 & 0.8 \end{pmatrix} \begin{pmatrix} N_j \\ N_{SA} \\ N_A \end{pmatrix}$$

We wanted to calculate the effects of different conservation strategies to the long-term-growthrate of the turtle-population; defined by:

$$e_{ij} = \frac{\partial \lambda}{\partial a_{ij}} \frac{a_{ij}}{\lambda} = v_i u_j \frac{a_{ij}}{\lambda}$$

Currently $\lambda \approx 0.95$, so the population is declining

Calculating leading left and right eigenvectors u and v enables us to substitute the needed terms:

$$e_{13} = V_1 u_3 \frac{a_{13}}{\lambda} = \frac{1}{700} \frac{60}{0.95} = 0.09$$

$$e_{33} = V_3 u_3 \frac{a_{33}}{0.95} = \frac{1}{700} \cdot 400 \cdot 1 \frac{0.8}{0.95} = 0.481$$

We see that increasing adult survival is more efficient strategy for conservation.