Dependence logic
Problems 4
Tuesday 19.4.2011

1. Formulate a sentence ϕ of dependence logic such that for all finite structures $\mathcal{M}$:
$$\mathcal{M} \models \phi \Leftrightarrow |M| \text{ is odd.}$$

2. A graph is 2-colorable if its elements can be devided into two disjoint parts so that all edges are between elements of different parts. Formulate a sentence of dependence logic which is true in a graph iff the graph is 2-colorable.

3. Show that if $\models \phi \rightarrow \psi$, then $\phi \Rightarrow^* \psi$.

4. Write down the sentence $\tau_{1,\phi}$, where $\phi = \exists x_0 P(x_0)$. (The translation $\phi \mapsto \tau_{1,\phi}$ was defined in the lectures and in Theorem 6.2 of the course textbook on page 88.)

5. Let ϕ be a sentence of Σ_1^1 of the form

$$\phi := \exists R \psi,$$

where R is a k-ary relation symbol and ψ is first-order. Find a formula of Σ_1^1 that is logically equivalent to the formula $\forall x_n \phi$.

6. The Löwenheim-Skolem Theorem of first-order logic shows: If a sentence ϕ of FO is true in arbitrarily large finite models or is true in a model of infinite cardinality, then ϕ has models in all infinite cardinalities. Show that dependence logic satisfies the Löwenheim-Skolem Theorem.