Dependence logic

Problems 2

Tuesday 5.4.2011

1. Let ϕ be the formula $(=(x_0, x_1) \vee \neg x_0 = x_1)$. Show that for all $\mathcal{M}$ and $X \neq \emptyset$:

$$\mathcal{M} \models_X \neg\phi \Leftrightarrow \mathcal{M} \not\models_X \phi.$$

2. The Closure Test shows that if $Y \subseteq X$ and $\mathcal{M} \models_X \phi$ then $\mathcal{M} \models_Y \phi$, for all formulas ϕ, models $\mathcal{M}$, and teams X. Complete the inductive proof of this result by considering the cases where ϕ is of the form $\neg\psi$, $\psi \vee \theta$, and $\exists x_n \psi$.

3. Show that the following strong logical equivalences hold:

$$\begin{aligned} \phi &\equiv^* \phi \wedge \top \\ \neg(\phi \wedge \psi) &\equiv^* (\neg\phi \vee \neg\psi) \end{aligned}$$

4. Show that the following strong logical equivalences hold:

$$\begin{aligned} \forall x_n(\phi \wedge \psi) &\equiv^* (\forall x_n \phi \wedge \forall x_n \psi) \\ \neg\forall x_n \phi &\equiv^* \exists x_n \neg\phi \end{aligned}$$

5. Show the following non-equivalences:

$$\begin{aligned} \phi \vee \neg\phi &\not\equiv^* \top \\ \phi \vee \phi &\not\equiv^* \phi \end{aligned}$$

6. Show the following non-equivalences:

$$\begin{aligned} \phi \wedge \phi &\not\equiv^* \phi, \text{ but } \phi \wedge \phi \equiv \phi \\ (\phi \wedge \psi) \vee \theta &\not\equiv^* (\phi \vee \theta) \wedge (\psi \vee \theta) \end{aligned}$$