Overview of cooperation results with German colleagues in Inverse Problems

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The trilateral workshop on inverse problems
Helsinki, March 4-5, 2010

Personal information

Education:

1997–2004	Fudan University, Shanghai; Bachelor and Master in Applied Mathematics;
2004–2007	Johannes Kepler University (JKU) Linz; Ph.D. in Technical Mathematics.

Career:

2004–2010	Johann Radon Institute for Computational and Applied Mathematics (RICAM), Linz;
Sep. 2010–	Fudan University, Shanghai.

Multi-parameter regularization

Cooperation with

Prof. Tautenhahn, *University of Applied Sciences Zittau/Grolitz, Germany*;

Prof. Freeden, *Geomathematics Group, TU Kaiserslautern, Germany*.

Problem formulation and notations

Consider linear ill-conditioned systems

$$A_0 x = y_0$$

- $x^\dagger$ – unknown (generalized minimal norm) solution
- $A_0 \in \mathscr{L}(R^n, R^m)$ (generally $m \geq n$)
- (A_0, y_0) – error free data

Multi-parameter regularization

Ill-conditioning

1. y_δ – given noisy right hand side with $\|y_0 - y_\delta\|_2 \le \delta$;
2. A_h – given noisy system matrix with $\|A_0 - A_h\|_F \le h$.
3. $x_{\delta,h}$ – (generalized minimal norm) solution of $A_h x = y_\delta$

$$\left.\begin{array}{c}\|y_0 - y_\delta\|_2 \le \delta \\ \|A_0 - A_h\|_F \le h\end{array}\right\} \not\Rightarrow \|x^\dagger - x_{\delta,h}\| \le \varepsilon(\delta,h); \quad \varepsilon(\delta,h) \to 0 \text{ as } \delta,h \to 0.$$

Regularized Total Least Squares - RTLS

Introduce total least squares (TLS) in a constraint setting as follows

$$\min_{x,y,A}\{\|A-A_h\|_F^2+\|y-y_\delta\|_2^2\} \quad \text{subject to} \quad \begin{array}{c} y=Ax, \\ \|Bx\|_2\le R. \end{array}$$

[Golub, Hansen and O'Leary 1999]

Dual RTLS

Introduce TLS in another constraint setting as follows:

$$\min_{x,y,A}\|Bx\|_2^2 \quad \text{subject to} \quad \begin{array}{c} y=Ax, \\ \|y-y_\delta\|_2\le\delta, \\ \|A-A_h\|_F\le h. \end{array}$$

Theorem 1 (DRTLS general case $B \neq I$)

Consider the DRTLS problem

$$\min_{x,y,A} \|Bx\|_2^2 \quad \text{subject to} \quad \begin{aligned} y &= Ax, \\ \|y - y_\delta\|_2 &\leq \delta, \\ \|A - A_h\|_F &\leq h. \end{aligned}$$

If the constraints are active, then the solution $x_{DRTLS} = x_{\alpha,\beta}^{\delta,h}$ can be obtained from the following equation

$$(A_h^T A_h + \alpha B^T B + \beta I)x = A_h^T y_\delta,$$

where (α, β) obeys

$$\|A_h x_{\alpha,\beta}^{\delta,h} - y_\delta\|_2 = \delta + h\|x_{\alpha,\beta}^{\delta,h}\|_2 \quad \text{and} \quad \beta = -h^2 - \frac{h\delta}{\|x_{\alpha,\beta}^{\delta,h}\|_2}.$$

Error bounds estimation

- X, Y - Hilbert spaces
- $A_0 \in \mathscr{L}(X,Y)$ with non-closed range $\mathscr{R}(A_0)$
- B - strictly positve self-adjoint (unbounded) operator in X

Assumption A1 (Link condition between A_0 and B^{-1})

$$m\|B^{-a}x\| \leq \|A_0 x\| \quad \text{for some} \quad a > 0, m > 0.$$

Assumption A2 (Solution smoothness)

$$x^\dagger \in M_{B,E} = \{x \in X : \|B^p x\| \leq E\} \quad \text{for some } p > 0.$$

Error bounds estimation

Theorem 2 (Order optimality for the RTLS method)

$$\left.\begin{array}{c}\text{Assumptions } A1, A2\\ p \in [1, 2+a]\\ R = \|Bx^\dagger\|\end{array}\right\} \rightarrow \|x_{RTLS} - x^\dagger\| = O\left((\delta + h)^{\frac{p}{p+a}}\right)$$

Theorem 3 (Order optimality for the DRTLS method)

$$\left.\begin{array}{c}\text{Assumptions } A1, A2\\ p \in [1, 2+a]\end{array}\right\} \rightarrow \|x_{DRTLS} - x^\dagger\| = O\left((\delta + h)^{\frac{p}{p+a}}\right)$$

References

LS, Pereverzev S. V. and Tautenhahan U. (2009)
Regularized total least squares: computational aspects and error bounds
SIAM J. Matrix Anal. Appl. 31, 918-941.

LS, Pereverzev S. V., Shao Y. and Tautenhahan U. (2010)
On the generalized discrepancy principle for Tikhonov regularization in Hilbert scales
To appear in Journal of Integral Equations and Applications.

LS and Pereverzev S. V. (2010)
Multiparameter regularization in downward continuation of satellite data
To appear in "Handbook of Geomathematics", Springer
(Edit by Freeden et.al.).

Pool boiling process - Heuristic parameter choice rules

Cooperation with

DI. Heng, Dr. Mhamdi,
AVT-Process System Engineering, RWTH Aachen, Germany;

PD. Dr. Mathé
Weierstrass Institute for Applied Analysis and Stochastics, Germany.

Pool boiling background

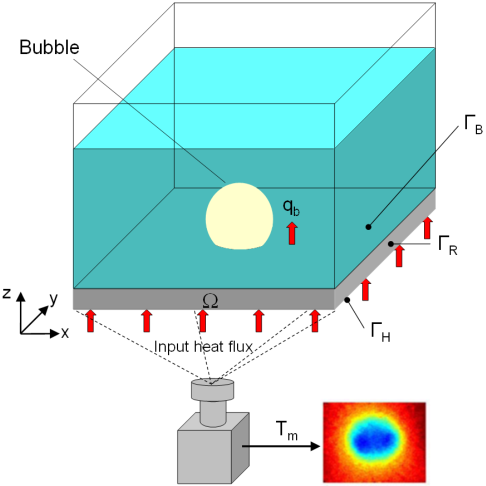
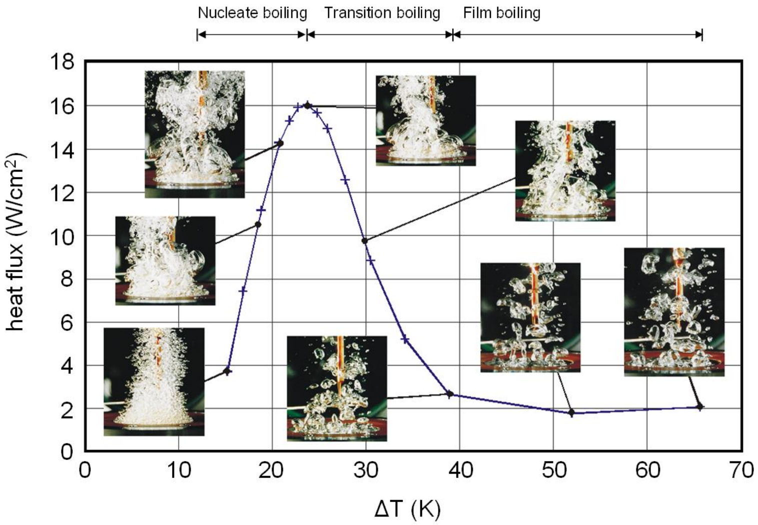


Figure: Schematic representation of a single-bubble nucleate boiling experiment.

Pool boiling background



Pool boiling background

Unmeasurable local heat flux on the boiling surface (up surface) = ?

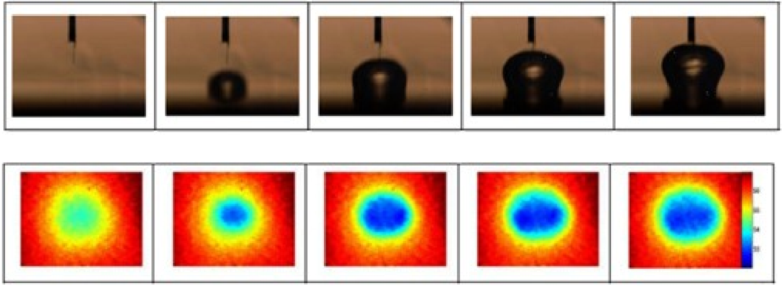


Figure: Upper: Direct observation of bubble;
Lower: Corresponding high-resolution temperature measurements on the heating surface (Lower boundary layer, TU Darmstadt)

Case study: real data

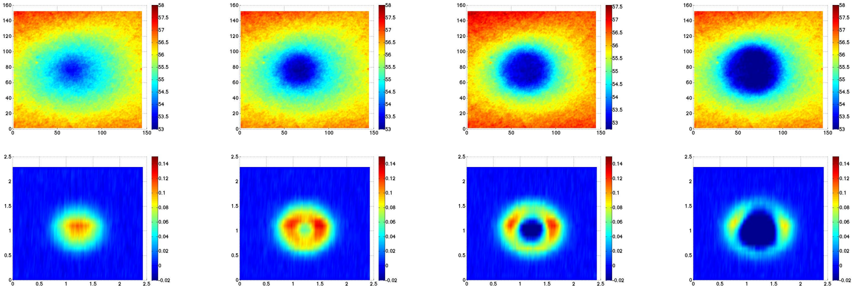


Figure: Upper: Measured temperature on the heating surface, Frame 24–27; Lower: Reconstructed heat flux on the boiling surface, Frame 24–27.

References

- Heng Y., LS, Mhamdi A. and Pereverzev S. V. (2009)
  Model function approach in the modified L-curve method for the choice of the regularization parameter
  RICAM report 2009-08, submitted
- LS and Mathe P. (2009)
  Heuristic parameter selection based on functional minimization: Optimality and model function approach
  WIAS report 1413, submitted

Future cooperation

- Ill-posed problems with noisy operators and right-hand sides, multiparameter regularizations;
- Regularization in Banach space, heuristic parameter choice rules for Banach space and different noise models;
- Numerical methods on ill-posed problems for PDEs.

Possible cooperation with Prof. Hofmann (TU Cheminitz) and some other experts in Inverse Problems from Finland or Germany.

Thank you for your attention!