

Solution 1

$H_0 : p_1 - p_2 = 0$
 $H_a : p_1 - p_2 > 0$
 T.S.

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{1370 \times 29\% + 1370 \times 18\%}{1370 \times 2} = \frac{643.9}{1370 \times 2} = 23.5\%$$

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1 - \hat{p})(\frac{1}{n_1} + \frac{1}{n_2})}}$$

$$= \frac{0.29 - 0.18}{\sqrt{0.235 \times 0.765 \times \frac{2}{1370}}}$$

$$\approx 6.8$$

R.R. Reject H_0 if $z > z_{.05} = 1.645$.

Conclusion:

Since $z_{ob} \gg 1.645$ we can reject H_0 at 5% level of significance and conclude that there has been a significant change in the percent of internet users who download music.

95% confidence interval for $p_1 - p_2$ is

$$\hat{p}_1 - \hat{p}_2 \pm z_{.05} \sqrt{\frac{\hat{p}_1(1 - \hat{p}_1)}{n_1} + \frac{\hat{p}_2(1 - \hat{p}_2)}{n_2}}$$

$$\implies (0.29 - 0.18) \pm 1.96 \times \sqrt{\frac{0.29 \times 0.71}{1370} + \frac{0.18 \times 0.82}{1370}}$$

$$\implies 0.11 \pm 0.03148$$

$$\implies (0.078516, 0.14148)$$

Solution 2

2 weeks				118	120		126	126	129	
16 weeks	98	110	110			124			140	
Rank	1	2.5	2.5	<u>4</u>	<u>5</u>	6	<u>7.5</u>	<u>7.5</u>	<u>9</u>	10

H_0 : Breaking strength have the same distribution.

H_a : Breaking strength are lower for strips buried longer.

T.S.

$$W = 4 + 5 + 7.5 + 7.5 + 9 = 33$$

$$\mu_W = \frac{5 \times (10 + 1)}{2} = 27.5$$

$$\sigma_W^2 = \frac{5 \times 5 \times (10 + 1)}{12} = 22.917$$

$$\sigma_W = 4.787$$

R.R. If $\frac{W - \mu_W}{\sigma_W} > 2$, reject H_0 .

Conclusion:

Since $\frac{W - \mu_W}{\sigma_W} = \frac{33 - 27.5}{4.787} = 1.1489 < 2$, we cannot reject H_0 .

Solution 3

Initial major	Area transferred to				Total
	Engineering	Management	Liberal arts	Other	
Biology	13 (25.3)	25 (34.6)	158 (130.2)	202 (207.9)	398
Chemistry	16 (7.2)	15 (9.9)	19 (37.3)	64 (59.6)	114
Mathematics	3 (4.6)	11 (6.3)	20(23.6)	38 (37.6)	72
Physics	9 (3.9)	5 (5.3)	14 (20.0)	33 (31.9)	61
Total	41	56	211	337	645

H_0 : Initial major and transferred area are independent.

H_a : Initial major and transferred area are NOT independent.

T.S.

$$\begin{aligned}\chi^2 &= \sum_i \sum_j \frac{(O_{ij} - E_{ij})^2}{E_{ij}} \\ &= \frac{(13 - 25.3)^2}{25.3} + \frac{(25 - 34.6)^2}{34.6} + \dots + \frac{(33 - 31.9)^2}{31.9} \\ &\approx 50.5 \\ \nu &= (4 - 1)(4 - 1) = 9\end{aligned}$$

R.R.

Reject H_0 if $\chi^2 > \chi_{.05}^2(9) = 16.92$

Conclusion:

Since $\chi_{ob}^2 > 16.92$, we can reject H_0 at 5% level of significance and conclude that there is dependence between initial major and transferred area.

Solution to 4

A	10	20	25	30	33	37	41	43	46	46						
r_A	1	2.5	4	6	7	9	11.5	13	14.5	14.5						
B		20	27			35	40	41			50	50	54	56	57	
r_B		2.5	5			8	10	11.5			16.5	16.5	18	19	20	

H_0 : Two teaching methods are the same.

H_a : Two teaching methods are not the same.

T.S.

$$\begin{aligned}W &= 1 + 2.5 + 4 + 6 + 7 + 9 + 11.5 + 13 + 14.5 + 14.5 = 83 \\ \mu_W &= \frac{10(20 + 1)}{2} = 5 \times 21 = 105 \\ \sigma_W^2 &= \frac{10 \times 10 \times (20 + 1)}{12} = 175 \quad \sigma_W = 13.23 \\ Z &= \frac{W - \mu_W}{\sigma_W} = \frac{83 - 105}{13.23} = -1.66 \\ \text{P-value} &= P(Z < -1.66) = 1 - 0.9515 = 0.0485\end{aligned}$$

Conclusion:

Since p-value is less than 5%, we can reject H_0 at 5% level of significance.

Solution to 5

d	1	1	2	2	2	3	3	3	3	3	3	6	6	6	6	6	(6)
R	1.5	1.5	4	4	4	8.5	8.5	8.5	8.5	8.5	8.5	14.5	14.5	14.5	14.5	14.5	14.5

$$W^+ = 138.5$$