

Please note that this is only an outline.

1.

$$\bar{X} = 35.09, \quad s^2 = 125.2008, \quad s = 11.189 \quad \alpha = 5\%$$

$$H_0 : \mu = 32$$

$$H_a : \mu > 32$$

$$T.S. : Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{35.09 - 32}{11.189/\sqrt{44}} = 1.83$$

$$\text{P-value} \quad P(Z > 1.83) = 1 - 0.9664 = 0.0336$$

Conclusion: Since the p-value is less than 5%, we can reject H_0 at 5% level of significance.

2.

$$H_0 : \mu = 0.3$$

$$H_a : \mu > 30$$

$$T.S. : Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{0.7 - 0.3}{0.4/\sqrt{60}} = 7.7459$$

$$\text{P-value} < 0.0001$$

Conclusion: reject H_0

3. (a)

$$\begin{aligned} \alpha &= P(\bar{X} > 43.12 | \mu_0 = 40) \\ &= P(Z > \frac{43.12 - 40}{60/\sqrt{1000}}) \\ &= P(Z > 1.64) \\ &= 1 - 0.9495 = 0.0505 \end{aligned}$$

(b)

$$\begin{aligned} \beta &= P(\bar{X} < 43.12 | \mu_a = 45) \\ &= P(Z < \frac{43.12 - 45}{60/\sqrt{1000}}) \\ &= P(Z < -0.99) \\ &= 1 - 0.8389 = 0.1611 \end{aligned}$$

(c)

$$\begin{aligned} \text{power} &= P(\bar{X} > 43.12 | \mu_a = 50) \\ &= P(Z > \frac{43.12 - 50}{60/\sqrt{1000}}) \\ &= P(Z > -3.63) = 1 \end{aligned}$$

4. $H_0 : \mu = 0$

$H_a : \mu > 0$

Rejection region:

For $\alpha = 5\%$

$$\bar{X} > \mu_0 + z_{.05} \frac{\sigma}{\sqrt{n}} = 0 + 1.645 \times \frac{2}{5} = 0.658$$

$$\text{Power of the test: } = P(X > 0.658 | \mu_a = 1) = P(Z > \frac{0.658-1}{2/\sqrt{25}}) = P(Z > -0.855) = 0.8051$$

For $\alpha = 10\%$

$$\bar{X} > \mu_0 + z_{.1} \frac{\sigma}{\sqrt{n}} = 0 + 1.28 \times \frac{2}{5} = 0.512$$

$$\text{Power of the test: } = P(X > 0.512 | \mu_a = 1) = P(Z > \frac{0.512-1}{2/\sqrt{25}}) = P(Z > -1.22) = 0.8888$$