Example 1

$\bar{x} = 41 \qquad s = \sqrt{\frac{1}{n-1}(\sum x_i^2 - \frac{(\sum x_i)^2}{n})} = 3.59$

$H_0: \quad \mu = 42$

$H_a: \quad \mu < 42$

T.S.

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{41 - 42}{3.59/\sqrt{10}} = -0.88 \qquad \nu = n - 1 = 9$$

R. R.: Reject H_0 if $t < -t_{0.05}(9) = -1.833$

Conclusion: Since observed $t_{ob} = -0.88 > -1.833$, we cannot reject H_0 and conclude that there is no sufficient evidence to support the conclusion that the mean life time of this brand of tires is less than 42, 000 miles.

Example 2

$H_0: \quad \sigma^2 = 0.6^2$

$H_a: \quad \sigma^2 \neq 0.6^2$

T.S.

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{(10-1) \times 0.060516}{0.6^2} = 1.5129 \qquad \nu = n - 1 = 9$$

R. R.: Reject H_0 if $\chi^2 > \chi^2_{.005}(9) = 2.36$ or $\chi^2 < \chi^2_{.995}(9) = 1.73$

Conclusion: Since $\chi^2 = 1.5129 < 1.73$, we can reject H_0 at 1% level of significance and conclude that the population variance of boxes of a cereal is not equal to 0.6^2.

90% confidence interval for σ^2 is

$$\left(\frac{(n-1)s^2}{\chi^2_{.05}(9)}, \frac{(n-1)s^2}{\chi^2_{.95}(9)} \right) \Longrightarrow \left(\frac{9 \times 0.060516}{16.9}, \frac{9 \times 0.060516}{3.33} \right) \Longrightarrow (0.032\,,\, 0.1636)$$

We are 90% certain that the true variance of boxes of cereals is in the interval (0.032 , 0.1636). Since this interval does not cover 0.6^2, we conclude that the sample of boxes of cereals is not selected from the population with variance 0.6^2. This result confirms conclusion made from hypothesis testing.

Example 3

$\bar{x_1} = 28.57 \qquad s_1^2 = 198.62$

$\bar{x_2} = 40 \qquad s_2^2 = 215.33$

Assume $\sigma_1 = \sigma_2$

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{6 \times 198.62 + 6 \times 215.33}{12}} \approx 14.39$$

$H_0: \mu_1 - \mu 2 = 0$

$H_a: \mu_1 - \mu_2 < 0$

T.S.

$$t = \frac{\bar{x_1} - \bar{x_2} - 0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{28.57 - 40}{14.39\sqrt{2/7}} = -1.49$$

R.R. For $\alpha = 0.05$, reject H_0 if $t < -t_{.05}(12) = -1.782$

Conclusion: Since observed $t = -1.49 > -1.782$, we cannot reject H_0. It means that we have not found sufficient evidence to support the alternative and there is no significant difference in the mean number of worms in the treated and untreated lambs.

Example 4

Let d_i be the difference between costs estimated at garage I and II.

d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}
0.3	1.1	1.1	-0.2	0.3	0.5	0.4	0.9	0.2	0.6	0.3	1.1	0.8	0.9	0.9

$$\bar{d} = 0.61 \qquad s_{\bar{d}} = \sqrt{\frac{1}{n-1}(\sum d_i^2 - \frac{(\sum d_i)^2}{n})} \approx 0.394$$

$H_0 : \mu_d = 0$
$H_a : \mu_d > 0$
T.S.

$$t = \frac{\bar{d} - 0}{s_{\bar{d}}/\sqrt{n}} = \frac{0.61}{0.394/\sqrt{15}} = 6 \qquad \nu = n - 1 = 14$$

Conclusion: Since p-value $P(t > 6)$ with $\nu = 14$ is less than 0.0005, we conclude that the mean repair estimation for garage I is greater than that for garage II.

Example 5

$H_0 : \sigma_1^2 = \sigma_2^2 \quad (\sigma_1^2/\sigma_2^2 = 1)$
$H_a : \sigma_1^2 \neq \sigma_2^2$
T.S.

$$F = \frac{s_1^2}{s_2^2} = 0.255 \quad \nu_1 = 9 \quad \nu_2 = 9$$

R.R. Reject H_0 if $F > F_{.025}(9, 9) = 4.03$
or if $F < F_{.975}(9, 9) = \dfrac{1}{F_{0.025}(9, 9)} = \dfrac{1}{4.03} = 0.248$
Conclusion: Since $0.248 < F_{\text{ob}} = 0.255 < 4.03$, we cannot reject H_0

Rule: $F_\alpha(\nu_1, \nu_2) = \dfrac{1}{F_{1-\alpha}(\nu_2, \nu_1)}$

Example 6 Test equal variance assumption in example 3

$H_0 : \sigma_1^2 = \sigma_2^2$
$H_a : \sigma_1^2 = \sigma_2^2$

$$s_1^2 = 198.62 \qquad s_2^2 = 215.33$$

T.S.

$$F = \frac{s_1^2}{s_2^2} = \frac{198.62}{215.33} = 0.9224$$

$$\nu_1 = n_1 - 1 = 6 \quad \nu_2 = n_2 - 1 = 6$$

R.R. Reject H_0 if $F > F_{0.025}(6, 6) = 5.82$
or $F < F_{.975}(6, 6) = \dfrac{1}{F_{.025}(6, 6)} = 0.1718$
Conclusion: Since $F_{\text{ob}} \in (0.1718, 5.82)$ we cannot reject H_0 at 5% level. Equal variances are assumed.

Example 8 See SPSS output

Example 9 See SPSS output

Example 10

$H_0 : p = 25\%$
$H_a : p \neq 25\%$
T.S.

$$z = \frac{\hat{p} - \hat{p_0}}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.32 - 0.25}{\sqrt{\frac{0.25 \times 0.75}{100}}} \approx 1.62$$

P-value $= 2P(Z > 1.62) = 0.105$
Conclusion: Since p-value $> 5\%$ we cannot reject H_0 at 5% level of significance.

95% confidence interval for population proportion p
$\hat{p} \pm z_{0.025}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
$\Longrightarrow 0.68 \pm 1.96\sqrt{\frac{0.68 \times 0.32}{100}}$
$\Longrightarrow (0.59,\ 0.77)$

We are 95% confident that 75% restaurant workers feel that work stress is damaging their personal life.

Example 11

$H_0 : p_1 - p_2 = 0$
$H_a : p_1 \neq p_2$ T.S.

$$\hat{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{1630 + 1684}{7180 + 9916} = 0.194$$

$$\begin{aligned} z &= \frac{\hat{p_1} - \hat{p_2}}{\sqrt{\hat{p}(1-\hat{p})(1/n_1 + 1/n_2)}} \\ &= \frac{0.227 - 0.17}{\sqrt{0.194 \times 0.806\big(1/7180 + 1/9916\big)}} \\ &\approx 9.34 \end{aligned}$$

Conclusion: p-value ≈ 0 indicating that population proportions of men and women frequent binge drinkers are not equal.

Example 12

The sign test for matched paired
Ignore pairs with difference 0; the number of trials is the count of the remaining pairs. The test statistic is the count X of pairs with a positive difference. P-values for X are based on the binomial $B(n, \frac{1}{2})$ distribution.

Let p be the probability that a randomly chosen dementia patient will have more aggressive behaviours on moon days than on other days. The null hypothesis of "no moon effect" says that the moon days are no different from other days, so a patient is equally likely to have more aggressive behaviors on moon days as on other days.
Let X be the number who have more aggressive behaviors on moon days.
$H_0 : p = \frac{1}{2}$
$H_a : p > \frac{1}{2}$
T.S.

$$X \sim \text{Bin}(15, \frac{1}{2}) \quad \text{under the} H_0$$

$$
\begin{aligned}
\text{P-value} &= P(X \geq 14) \\
&= P(X = 14) + P(X = 15) \\
&= 15\left(\frac{1}{2}\right)^{15} + \left(\frac{1}{2}\right)^{15} \\
&= 0.000488
\end{aligned}
$$

Conclusion: Since p-value is smaller than 0.0005, there is a very strong evidence in favor of an increase in aggressive behaviors on moon days.