Examples of confidence interval and statistical test

Example 1:
A tire company guarantees that a particular brand of tire has a mean useful lifetime of 42,000 miles or more. A consumer test agency, wishing to verify this claim, observed 10 tires on a test wheel that simulated normal road conditions. The lifetimes (in thousands of miles) were as follows:
42 36 46 43 41 35 43 45 40 39
Use these data to determine whether there is sufficient evidence to contradict the manufacturer's claim. Use $\alpha = 0.05$.

Example 2:
If 10 boxes of a cereal weighed: 10.2, 9.7, 10.1, 10.3, 10.1, 9.8, 9.9, 10.4, 10.3, 9.8 ounces, test the claim that σ is 0.6 against the claim that σ is not 0.6 at 0.01 level.
$\bar{x} = 10.06 \quad s = 0.246$
Again use these data and construct a 90% confidence interval for population variance of weight.

Example 3:
An experiment was conducted to compare the mean number of tapeworms in the stomachs of sheep that had been treated for worms against the mean number in those that were untreated. A sample of 14 worm-infected lamps was randomly divided into two groups. Seven were injected with the drug, and the remainder were left untreated. After a 6-month period, the lambs were slaughtered and the following worm counts were recorded:

Drug-treated sheep	18	43	28	50	16	32	13
Untreated sheep	40	54	26	63	21	37	39

Is the mean number of worms for treated lambs less than that for the untreated?

Example 4:
The insurance adjusters are concerned about the high estimates they are receiving from garage I for auto repairs compared to those from garage II. To verify their suspicions, each of 15 cars recently involved in an accident was taken to both garages for separate estimates of repair costs.

Car	Garage I	Garage II
1	7.6	7.3
2	10.2	9.1
3	9.5	8.4
4	1.3	1.5
5	3.0	2.7
6	6.3	5.8
7	5.3	4.9
8	6.2	5.3
9	2.2	2.0
10	4.8	4.2
11	11.3	11
12	12.1	11
13	6.9	6.1
14	7.6	6.7
15	8.4	7.5

Analyze this suspicion by proper statistics.

Example 5:

The life span of an electrical component was studied under two operating voltages, V_1 and V_2. Ten different components were randomly assigned to each of the two operating voltages. Use the following data to test at 5% whether $\sigma_1^2 / \sigma_2^2 = 1$ (σ_1^2 / σ_2^2 the ratio of the variances in life spans for the two populations, populations 1 and 2, corresponding to the components studies under V_1 and V_2, respectively.)

Voltage V_1: $n_1 = 10$, $s_1^2 = 0.51$

Voltage V_2: $n_2 = 10$, $s_2^2 = 0.20$

Example 6:

One major application of a test for the equality of two population variances is for checking the validity of the equal variance assumption for the two-sample t test.

Test in example 3, whether the variances of two populations are equal or not at 5% level of significance.

Example 7:

In the estimation of one population mean under the condition that the population variance is unknown, we can replace t statistic by z statistic (standard normal variable) when the sample size is sufficient large. Usually $n \geq 30$ is considered as sufficient large.

Hints:

Example 1: (Ott and Mendenhall understanding Statistics 6th, edition p269 example 7.5)

t statistic for one population mean when population variance is unknown and $n < 30$ *and population is assumed to be normally distributed.*

Example 2: χ^2 *statistic for population variances, under the assumptions that the population is normal.*

Example 3: (edited from Ott and Mendenhall understanding Statistics 6th, edition p299 e.g.8.3)

t statistic for the difference of two population means. Two populations are assumed to be independent normal distributions. Variances of the two populations are unknown.

Example 4: (Ott and Mendenhall understanding Statistics 6th, edition p312 example 8.7)

t statistics for the difference of two population mean. Mean difference is assumed to be normally distributed and $n < 30$. *Two populations are dependent.*

Example 5: (edited from Ott and Mendenhall understanding Statistics 6th, edition p369 e.g. 9.5)

F statistic for two population variances under the assumption that two populations are normal and independent.

Paired T tests

The following are the key points to remember concerning matched pairs:

1. A matched-pair analysis is needed when there are two measurements or observations on each individual and we want to examine the difference.
2. For each individual, use the difference between the two measurements as the data for your analysis. Difference is normally distribution.
3. Use one-sample t statistic for confidence interval estimation and hypothesis testing.

Pooled variance T test and separate variance T test

1. Two samples are selected randomly from two independent normal populations. We can make a rough judgement about normality of the population distribution by drawing histogram, stem-leaf-plot, box-whisker plot or normal quantile plot (QQ plot).
2. If the above assumptions are satisfied, construct an F test to test the equality of the two population variances.
3. If F test result shows equal variance, we use pooled variance T test, and if not, we use separate variance T test. Most of the statistical software reports these two results along with F test.
4. Both the T statistics and F statistics are sensitive to normality assumption. They should be used with caution unless the samples are large.

Example 8:

An educator believes that new directed reading activities in the classroom will help elementary school pupils improve some aspects of their reading ability. She arranges for a third-grade class of 21 students to take part in these activities for an eight-week period. A control classroom of 23 third-grades follows the same curriculum without the activities. At the end of the eight weeks, all students are given a Degree of Reading Power (DRP) test, which measures the aspects of reading ability that the treatment is designed to improve. The summarized statistics are

Group	n	$\bar{x}$	s
Treatment	21	51.48	11.01
Control	23	41.52	17.15

Do reading activities improve the reading ability?

Example 9:

Many people believe that the moon influences the actions of some individuals. A study of dementia patients in nursing homes recorded various types of disruptive behaviours every day for 12 weeks. Days were classified as moon days if they were in a three day period centered at the day of the full moon. For each patient the average number of disruptive behaviours was computed for moon days and for all other days. The data for the 15 subjects whose behaviours were classified as aggressive are presented in as below. The patients in this study are not a random sample of dementia patients. However, we examine their data in the hope that what we find is not unique to this particular group of individuals and applies to other patients who have similar characteristics.

Patient	Moon days	Other days	difference
1	3.33	0.27	3.06
2	3.67	0.59	3.08
3	2.67	0.32	2.35
4	3.33	0.19	3.14
5	3.33	1.26	2.07
6	3.67	0.11	3.56

7	4.67	0.3	4.37
8	2.67	0.4	2.27
9	6	1.59	4.41
10	4.33	0.6	3.73
11	3.33	0.65	2.68
12	0.67	0.69	-0.02
13	1.33	1.26	0.07
14	0.33	0.23	0.10
15	2	0.38	1.62

To assess whether there is a difference in aggressive behaviours on moon days versus other days.

Example 10:
A sample of 100 restaurant workers were asked whether or not work stress had a negative impact on their personal lives and 32 of them responded "No." A large national survey reported that 25% of workers reported a no negative impact. Test whether the result of the national survey is true according to this sample.

68% worker in this sample agreed that work stress had a negative impact on their personal lives. Estimate the corresponding population proportion by a 95% confidence interval.

Example 11:
Are men and women college students equally likely to be frequent binge drinkers? We examine the survey data below:

Population	n	X	$\hat{p} = X/n$
1 men	7,180	1,630	0.227
2 women	9,916	1,684	0.17
Total	17,096	3,314	0.194

The sample proportions are certainly quite different, but we will perform a significance test to see if the difference is large enough to lead us to believe that the population proportions are not equal.

Inference for nonnormal populations

1. Skewness is the chief barrier to the use of t procedures on the data without outliers. We can attempt to transform skewed data so that the distribution is symmetric and as close to normal as possible. The commonly used transformation function is logarithm or log, or power transformation.
2. Another strategy is to use a distribution-free inference procedure. Such procedures do not assume that the population distribution has any specific form, such as normal. Distribution-free procedures are often called nonparametric procedures.
3. In some cases a distribution other than a normal distribution will describe the data well. Choose non normal models for the data.

Example 12:
Sign test on example 9