Introduction to LaTeX
Exercises 2 (Group 6)

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30th March 2011

The `.tex` file containing your solutions to this exercise sheet should be emailed to `clifford.gilmore@helsinki.fi` before 15:00 on 5th April. The produced document should contain enough text to fill two pages. If you can't think of anything to write then you can find random text from Lorem Ipsum at http://www.lipsum.com/

1. Create a document titled, *LaTeX Solutions 2*, with you as the author.
2. Create a section titled *Miscellaneous Mathematics* and reproduce the following text in this section (it does not have to be in an enumerated list):

   (a) Pythagoras states for a right angled triangle with side lengths a, b, c, then $a^2 + b^2 = c^2$.

   (b) Euler's identity states that

   $$e^{i\pi} + 1 = 0$$

   (c)

   $$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{k^2} = \frac{\pi^2}{6} \tag{1}$$

   (d) Pascal's rule is

   $$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1} \tag{2}$$

   (you will need the amsmath package for this)

   (e)

   $$\int_0^{\frac{\pi}{2}} \cos x \mathrm{d}x$$

3. Create a section titled, *Real and Fourier Analysis.* Define your own theorem structure with the `newtheorem` command and then use it to state the following theorem. (Hint: use the `align*` environment to align the equations)

   Let $K = \{(x, y) \in \mathbb{R} : 0 \leq x \leq 1 \text{ and } 0 \leq y \leq x\}$ and let $f : K \to \mathbb{R}$ be an integrable function. Then

$$\begin{aligned} \int_K f dm_2 &= \int_{[0,1]} \left(\int_{[0,x]} f(x, y) dm_1(y) \right) dm_1(x) \\ &= \int_{[0,1]} \left(\int_{[y,1]} f(x, y) dm_1(x) \right) dm_1(y) \end{aligned}$$

4. Using the `proof` environment (accessed through the `amsthm` package), add a proof for the above theorem. (You don't need to give the real proof, any paragraph of text that is five lines long will do)

5. Add a subsection called *Fourier Analysis* to this section and reproduce the below text in your document. Note the question number will be generated automatically by LaTeX and may not necessarily be the number **2.1**. (Hint: using the `newtheorem` command, define a *Question* structure for the statement and use the `eqnarray*` environment to align the calculations)

   Question 2.1. *Let $f(\theta) = |\theta|, \theta \in [-\pi, \pi]$. Prove that $\hat{f}(0) = \frac{\pi}{2}$ and*

$$\hat{f}(n) = \frac{-1 + (-1)^n}{\pi n^2}, \qquad n \neq 0.$$

   Proof. It is easily seen that

$$\hat{f}(0) = \frac{1}{2\pi} \int_{-\pi}^{\pi} |\theta| \; \mathrm{d}\theta = \frac{1}{\pi} \int_0^{\pi} \theta \; \mathrm{d}\theta = \frac{\pi}{2}.$$

Next, for non-zero integers n we have,

$$\begin{aligned}
\hat{f}(n) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} |\theta| e^{-in\theta}\ \mathrm{d}\theta \\
&= \frac{1}{\pi}\int_{0}^{\pi} \theta\cos n\theta\ \mathrm{d}\theta \\
&= \frac{1}{\pi}\left(\left[\frac{\theta\sin n\theta}{n}\right]_0^\pi - \frac{1}{n}\int_0^\pi \sin n\theta\ \mathrm{d}\theta\right) \\
&= -\frac{1}{n\pi}\left[-\frac{\cos n\theta}{n}\right]_0^\pi \\
&= \frac{\cos n\pi - 1}{\pi n^2} \\
&= \frac{(-1)^n - 1}{\pi n^2} = \begin{cases} 0 & \text{if } 2 \mid n, \\ -\frac{2}{\pi n^2} & \text{if } 2 \nmid n. \end{cases}
\end{aligned}$$

□