Structure of tomographic X-ray data and examples of 3D imaging

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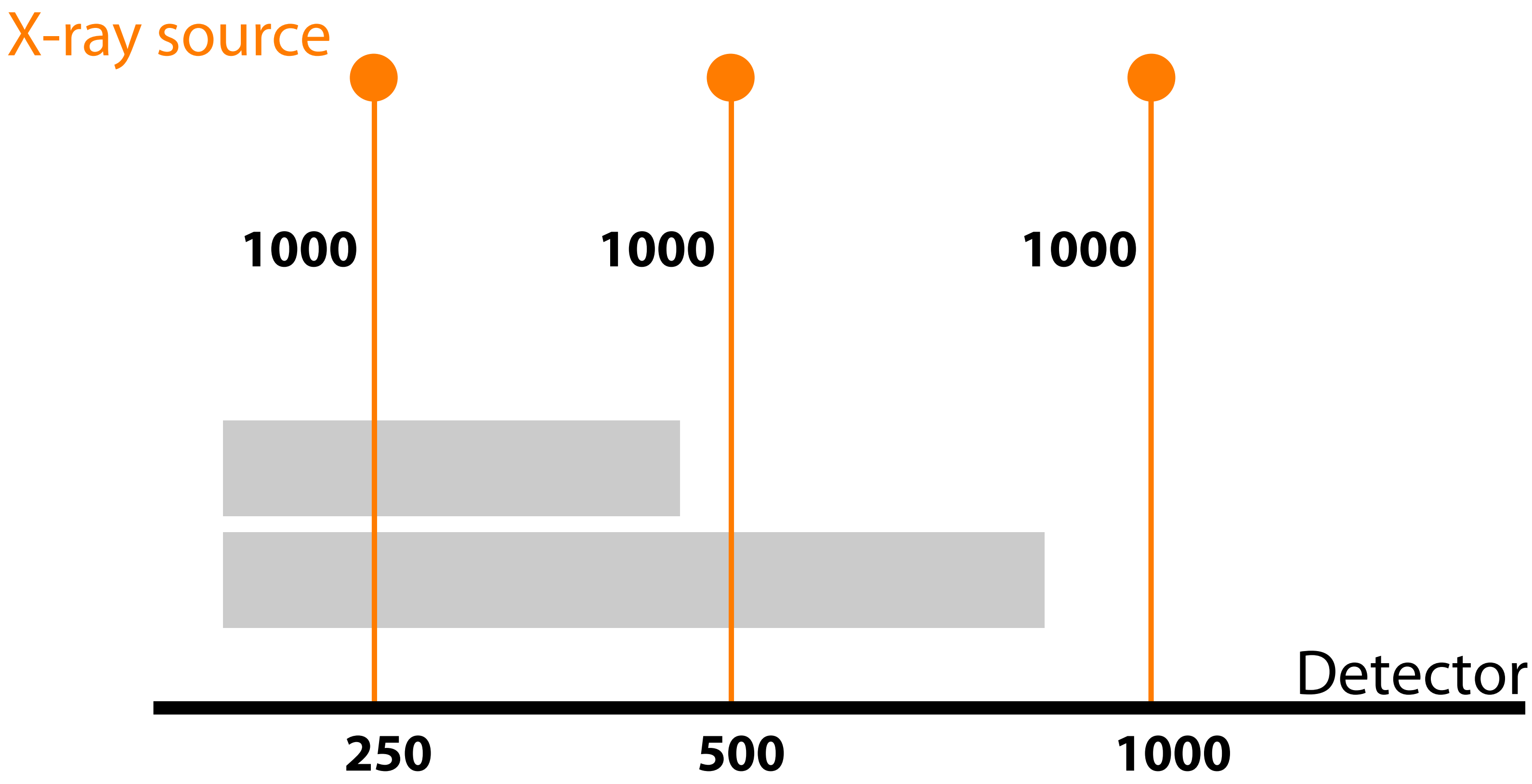
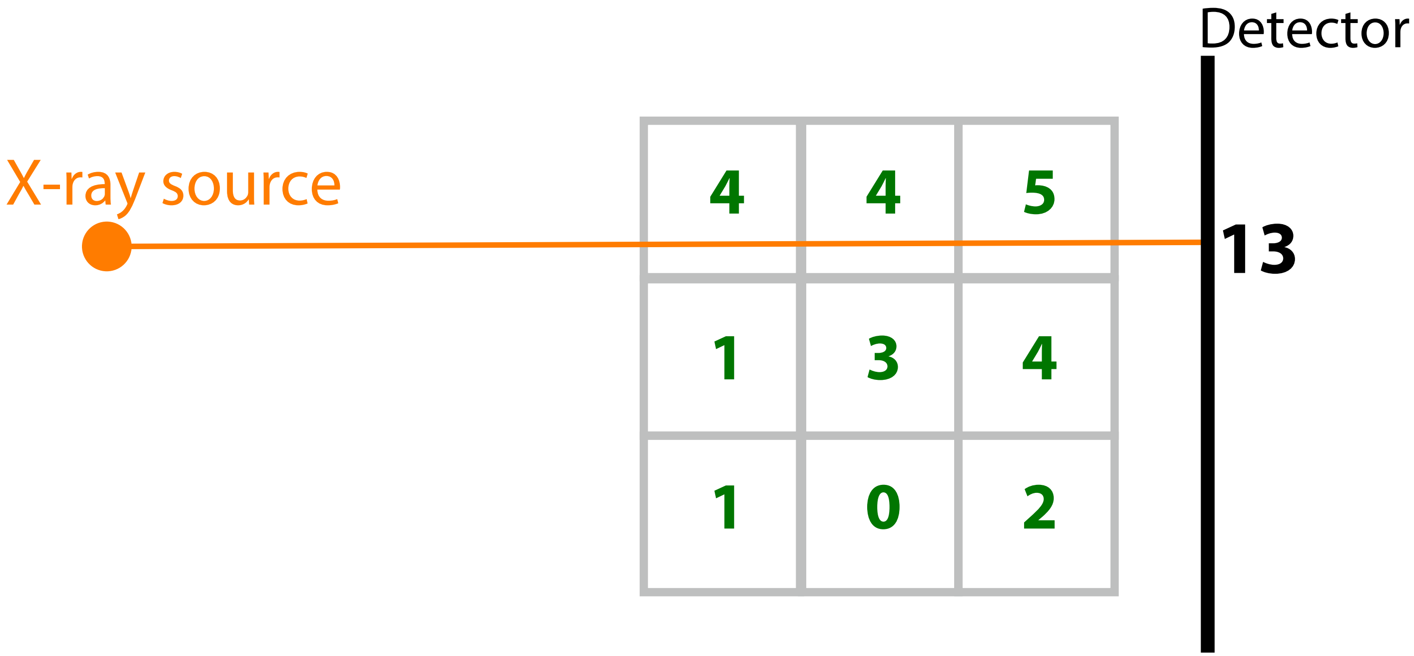
Course on computational inverse problems

X-ray images as measurements

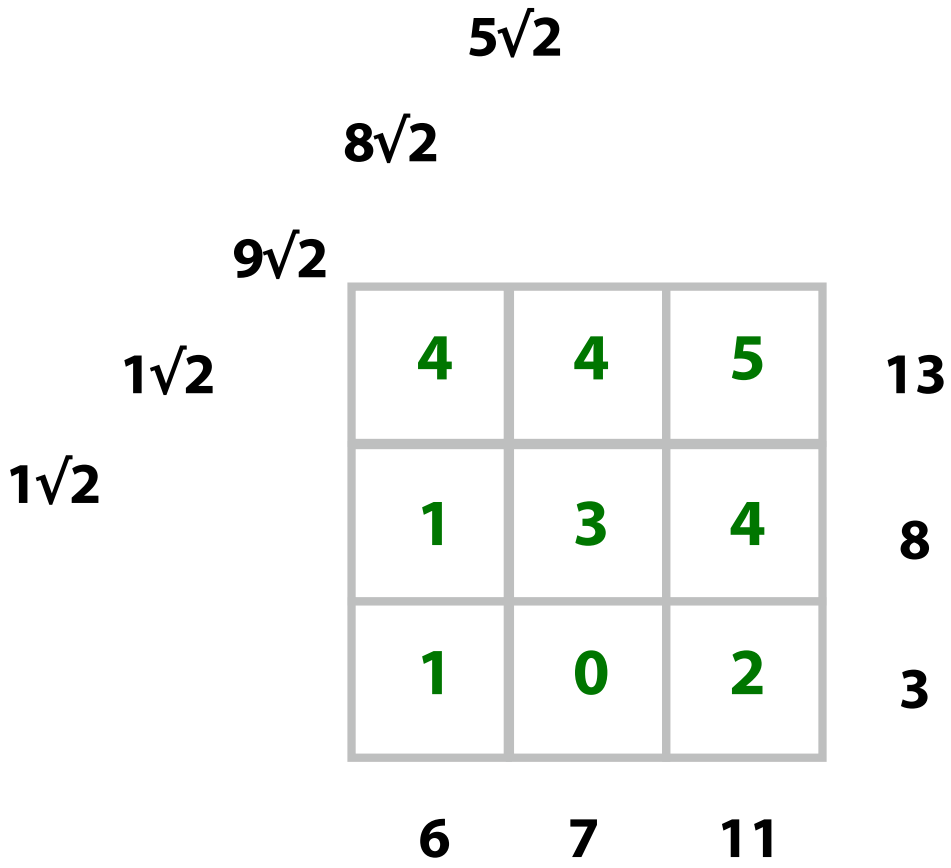


Logarithm	5,5	6,2	6,9
Density	1,4	0,7	0,0

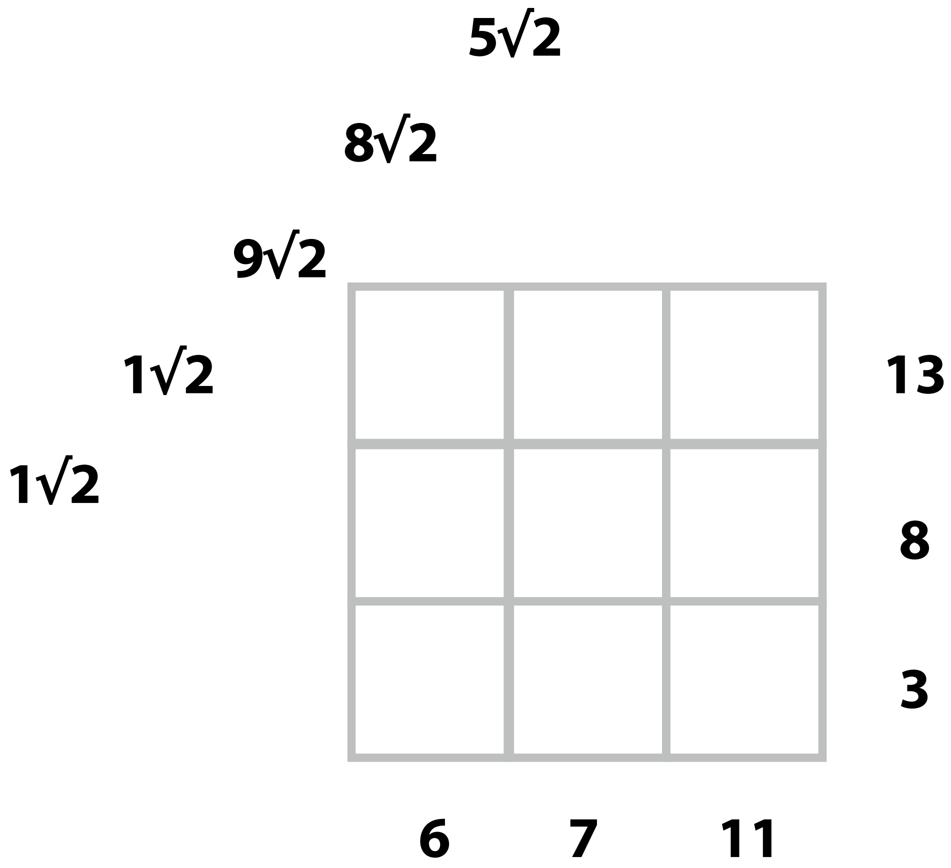
Every X-ray measures the sum of attenuation through tissue



Direct problem of tomography is to find the radiographs from given tissue

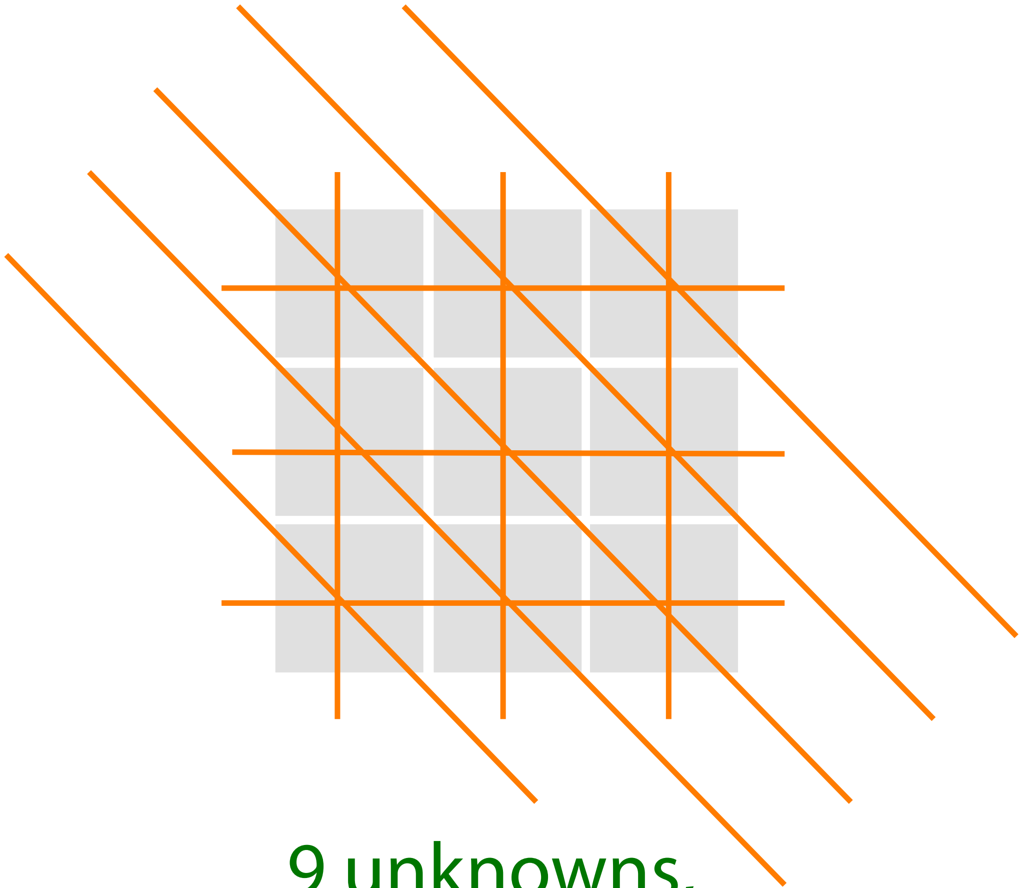


Inverse problem of tomography is to find the tissue from radiographs

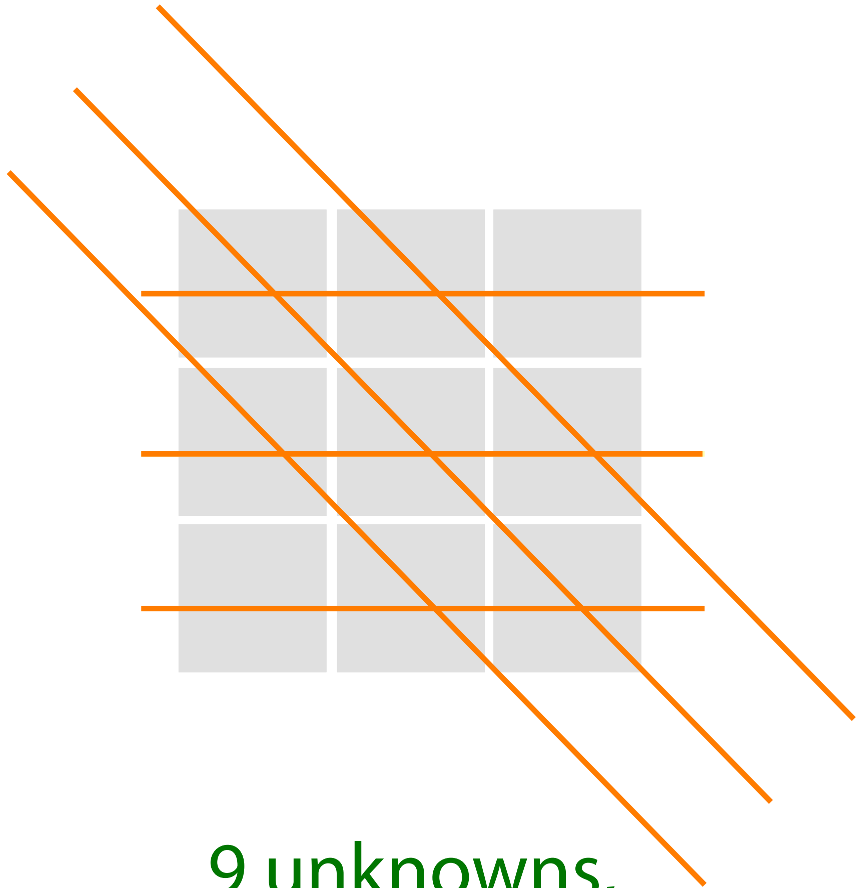


9 unknowns, 11 linear equations

The limited angle problem is harder than the full angle problem



9 unknowns,
11 linear equations



9 unknowns,
6 linear equations

In limited angle 3D imaging there are many tissues matching the radiographs

8√2

9√2

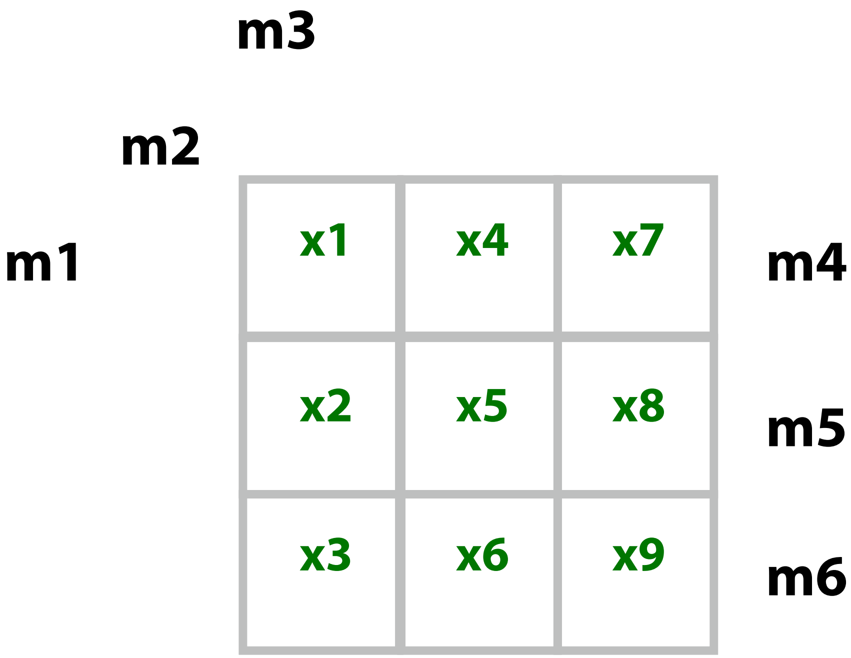
1√2				
	4	4	5	13
	1	3	4	8
	1	0	2	3

5	6	2
1	5	2
4	0	-1

9	1	3
1	0	7
3	0	0

a priori information is needed!

We write the reconstruction problem in matrix form and assume Gaussian noise



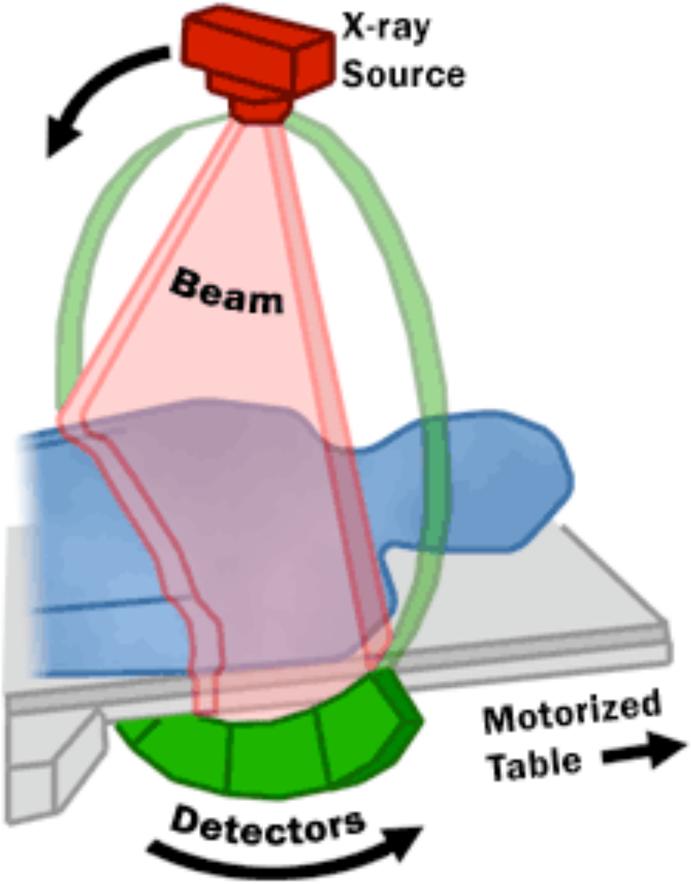
$$x = [x_1, x_2, \ldots, x_9]^T$$
$$m = [m_1, m_2, \ldots, m_6]^T$$

$$Ax = m$$

We consider the measurement model m=Ax+ε
with additive Gaussian noise ε of standard deviation σ.

To explain low-dose 3D imaging, let's start by discussing traditional 3D imaging

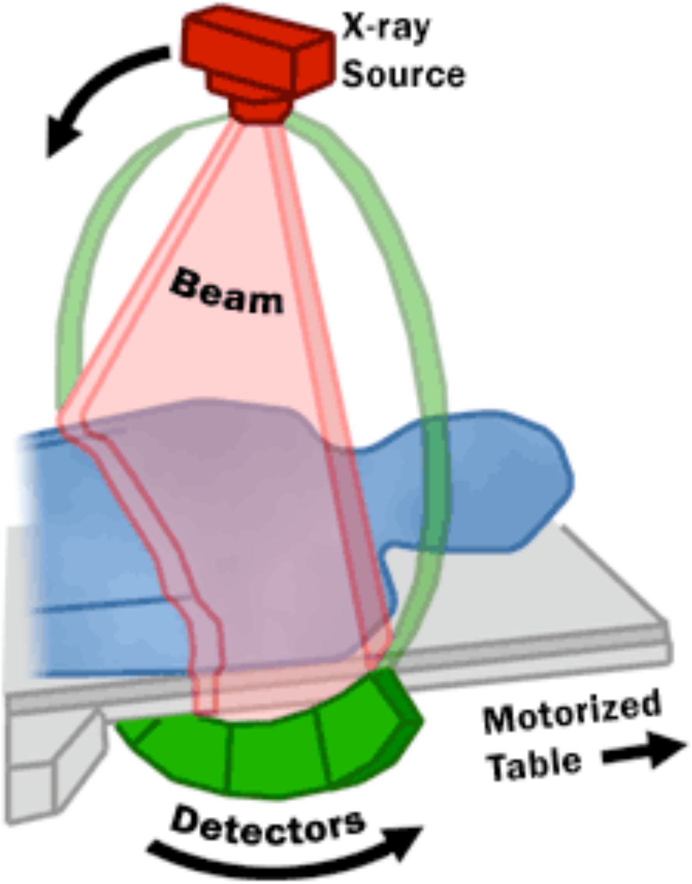
X-ray attenuation data is collected from 180 directions separately for each two-dimensional slice.

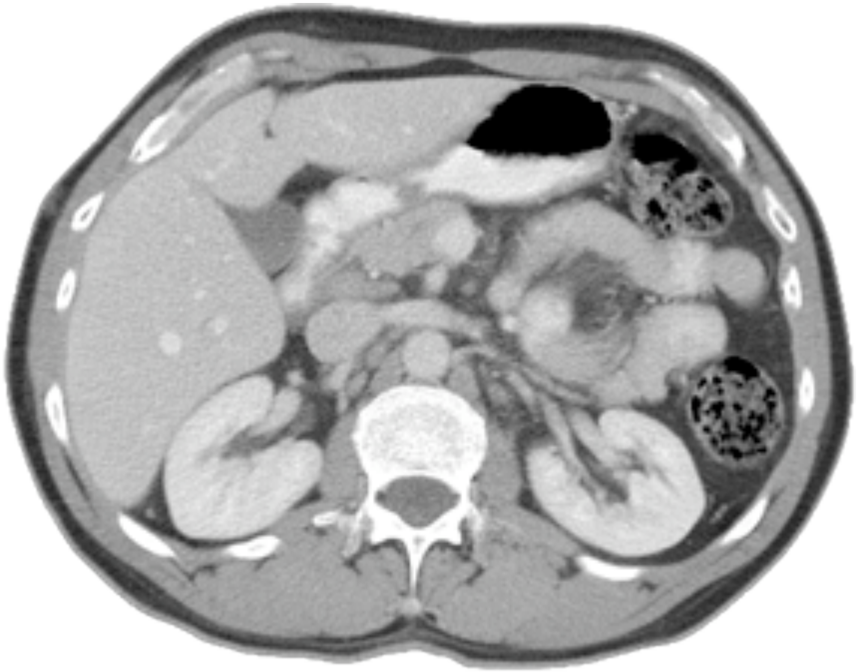


Images from http://www.fda.gov/cdrh/ct/what.html

Using a reconstruction algorithm, inner structure in the slice is revealed

This is called computerized tomography (CT).





Reconstruction of a function from its line integrals was first invented by

Johann Radon (1887-1956).

This is the famous inversion formula from 1917
for the Radon transform Rf of a function f:

$$f(x) = \frac{1}{4\pi^2} \int_{S^1} \int_{\mathcal{R}} \frac{\frac{d}{ds}(Rf)(\theta, s)}{x \cdot \theta - s} ds d\theta$$

Filtered back-projection

$$f(x) = \frac{1}{4\pi^2}\int_{S^1}\int_{\mathcal{R}} \frac{\frac{d}{ds}(Rf)(\theta,s)}{x\cdot\theta - s}\,ds\,d\theta$$

Filtered back-projection (FBP) is mathematical technology used on a daily basis in hospitals around the world.

The quality of 3D reconstruction using FBP is excellent.

Nobel prize was awarded to Hounsfield and Cormack 1979.

However, a comprehensive data set is mandatory for FBP.

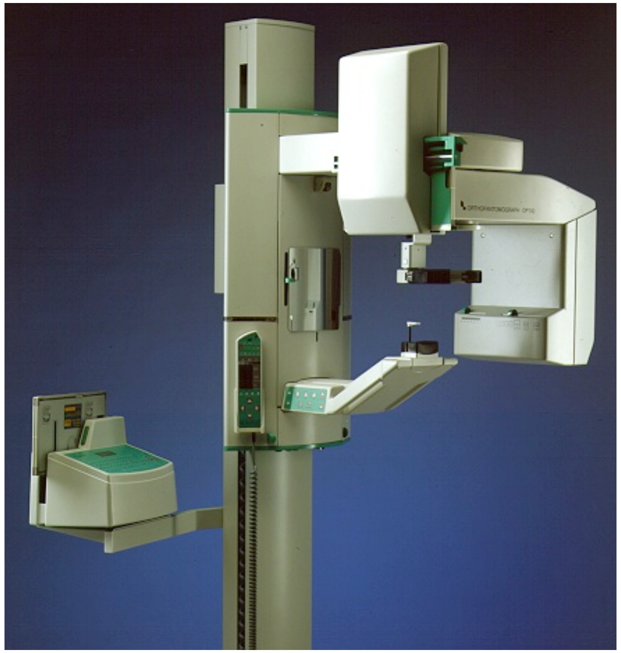
A series of projects started in 2001 aiming for a new type of low-dose 3D imaging

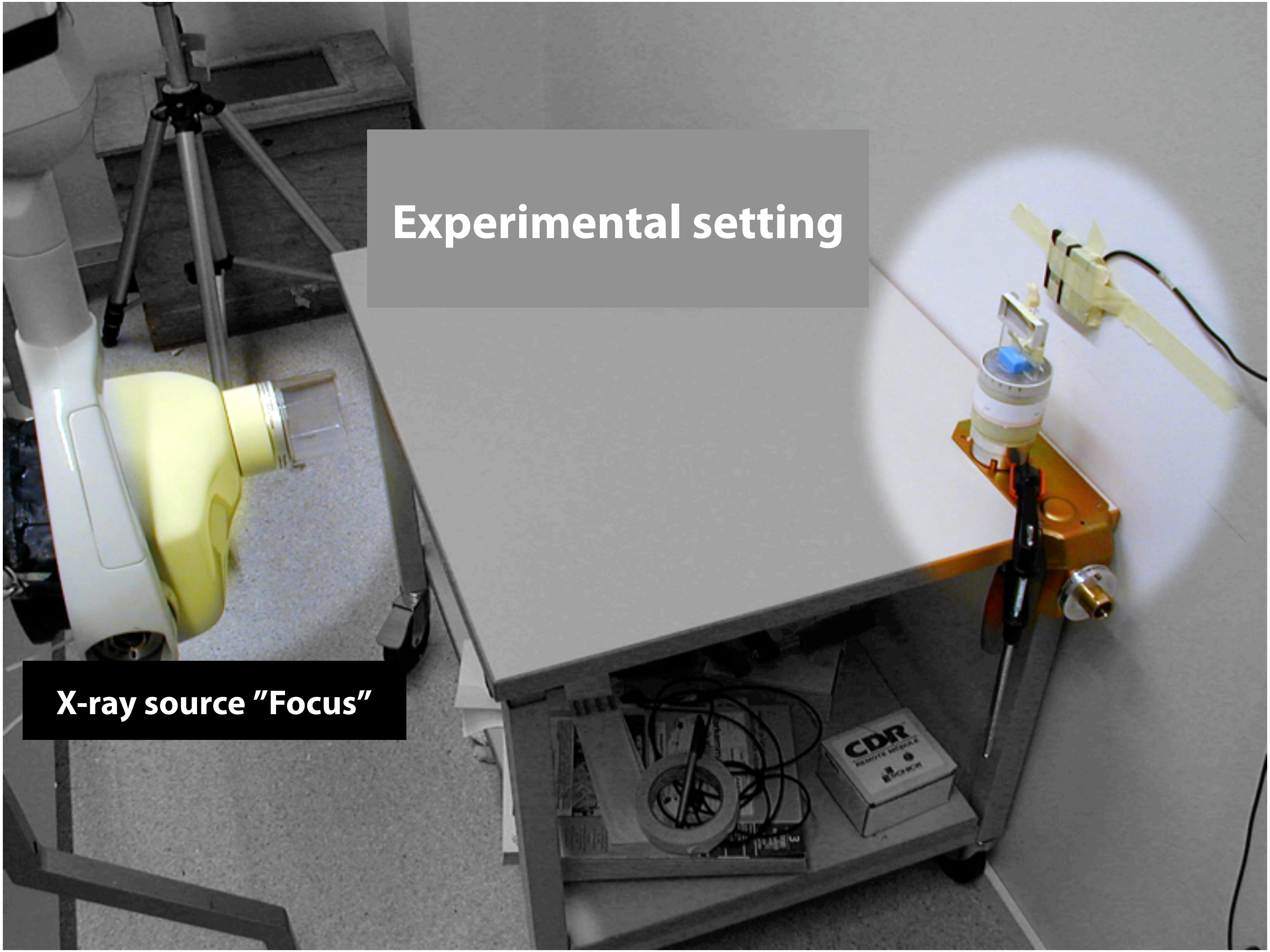
The goal was a mathematical algorithm with

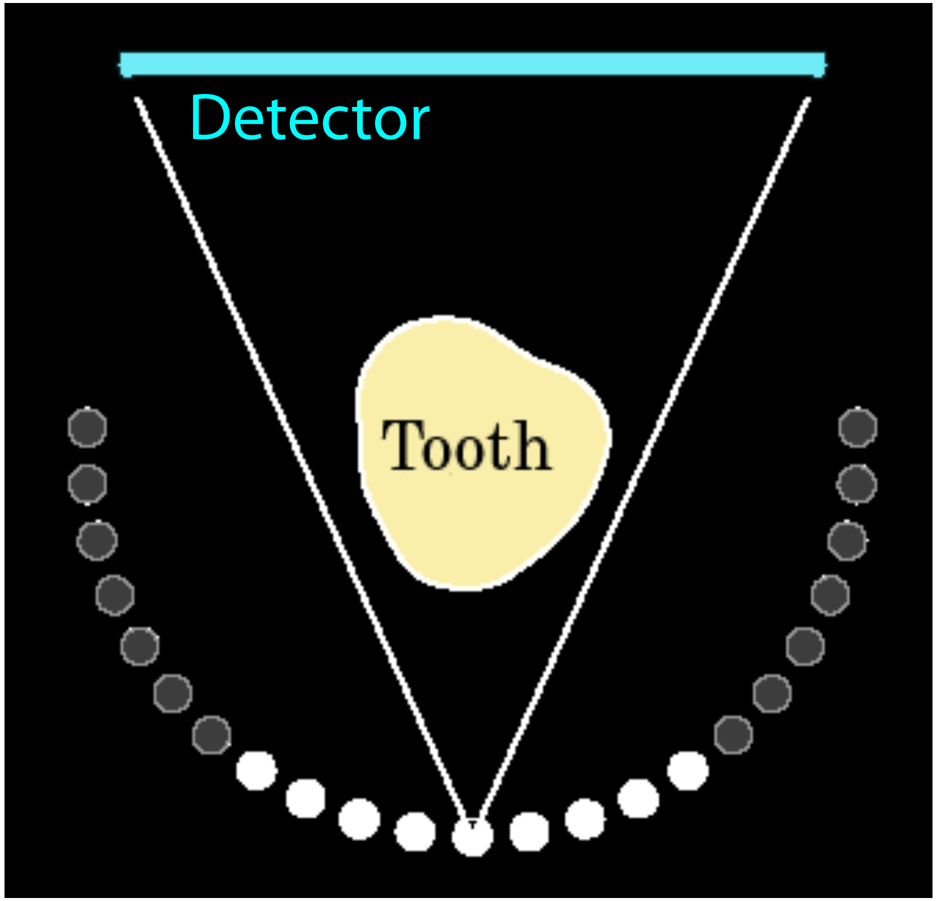
Input: small number of digital X-ray images taken with any X-ray device

Output: three-dimensional reconstruction with quality good enough for the clinical task at hand

Products of Instrumentarium Imaging in 2001:

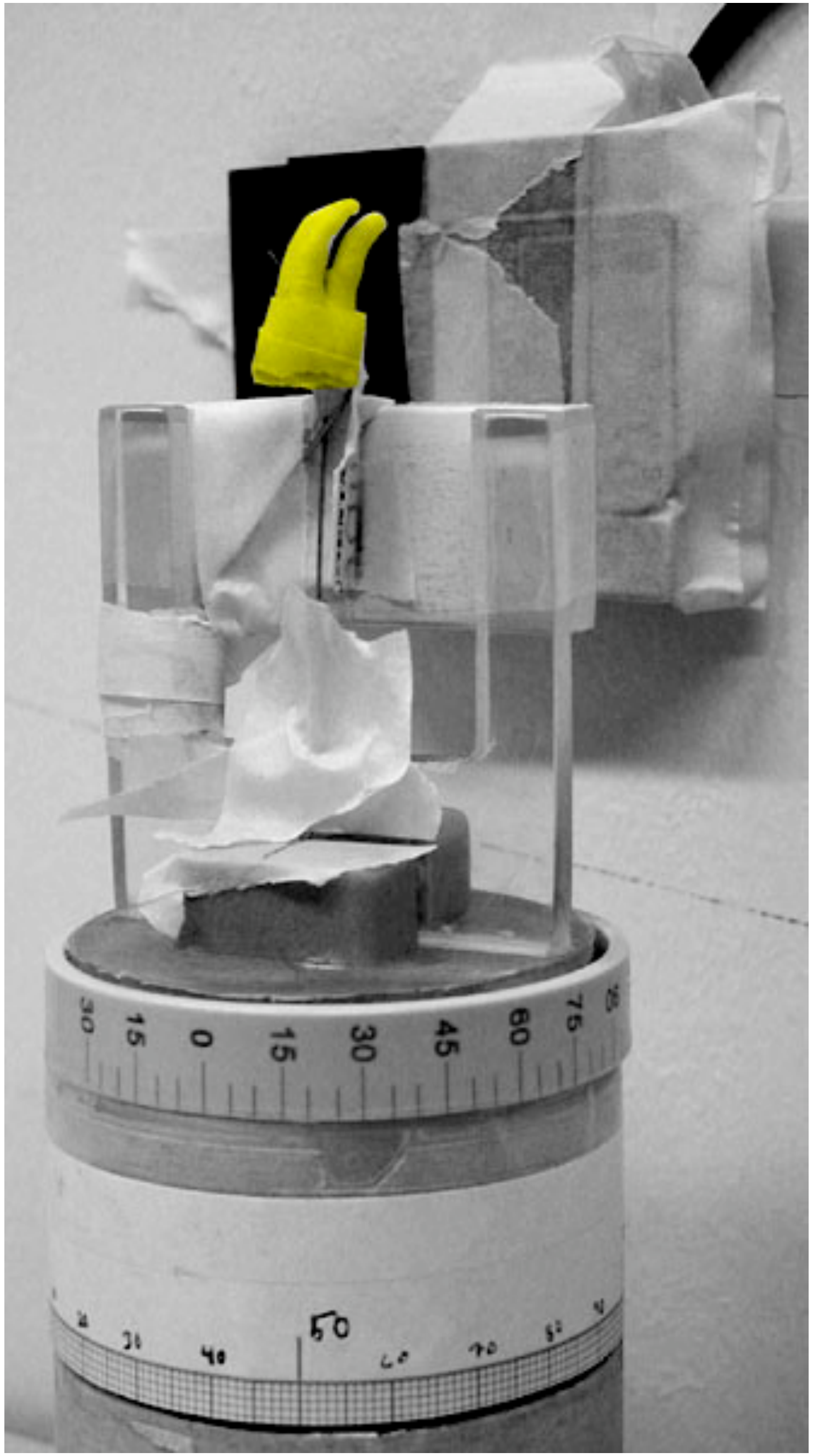



Experimental setting
X-ray source ”Focus”
CDR

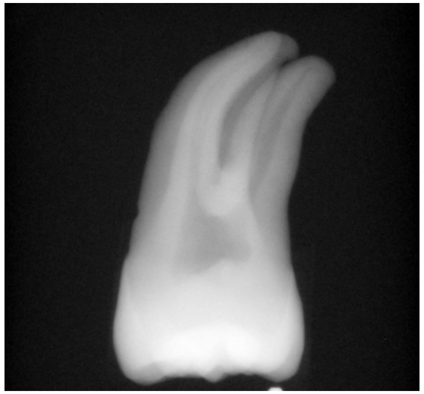


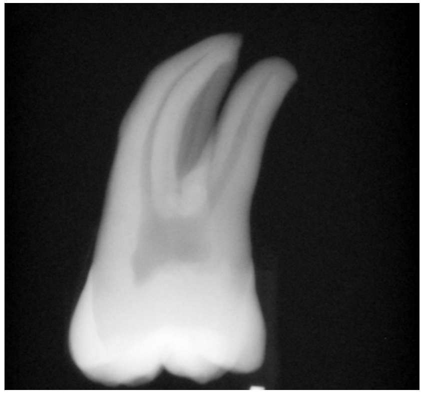
X-ray source positions

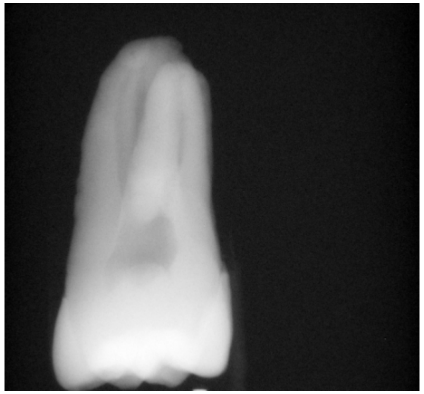
Tooth donated to science
by Helena Sarlin. Thanks!



The projection images look like this

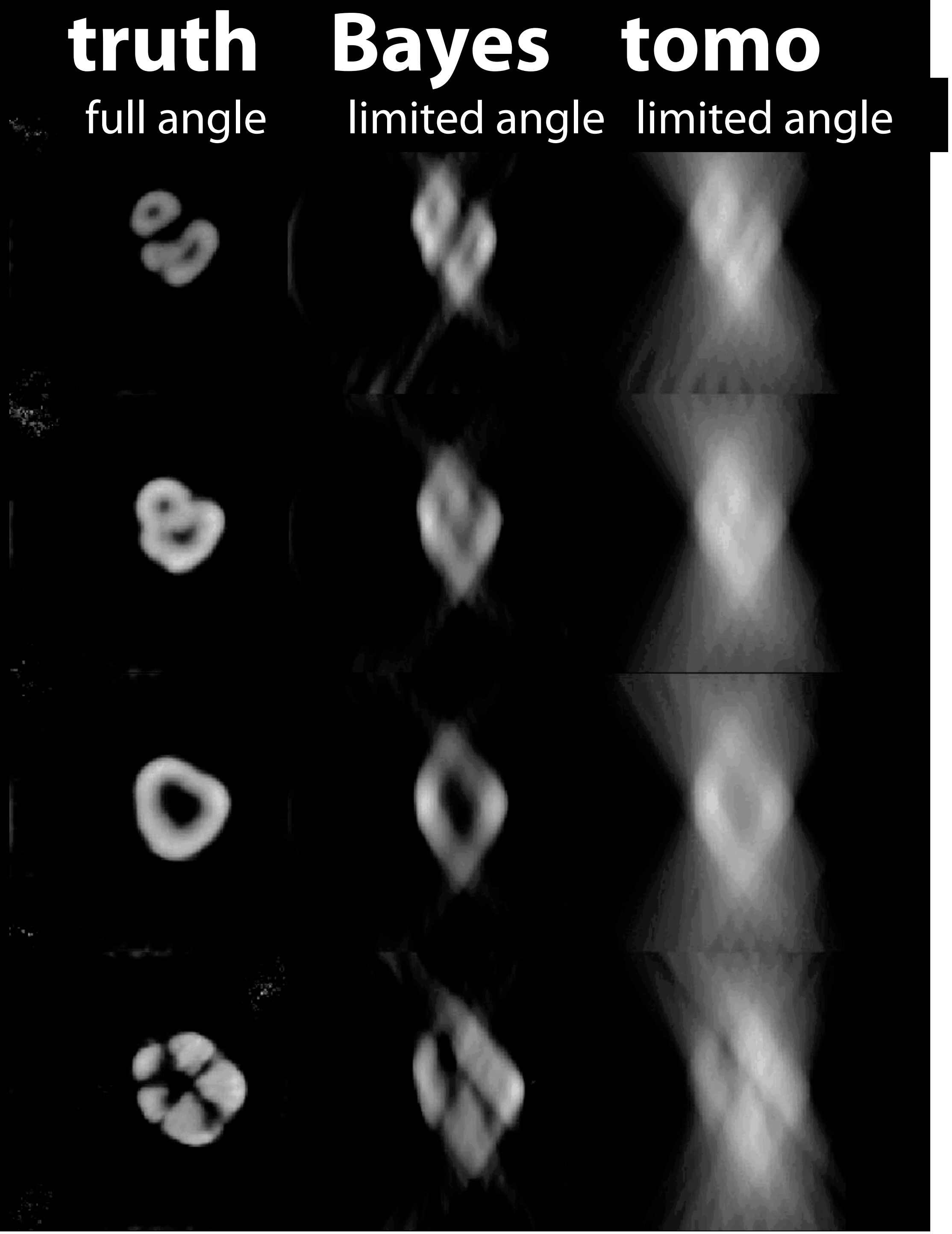

0


30

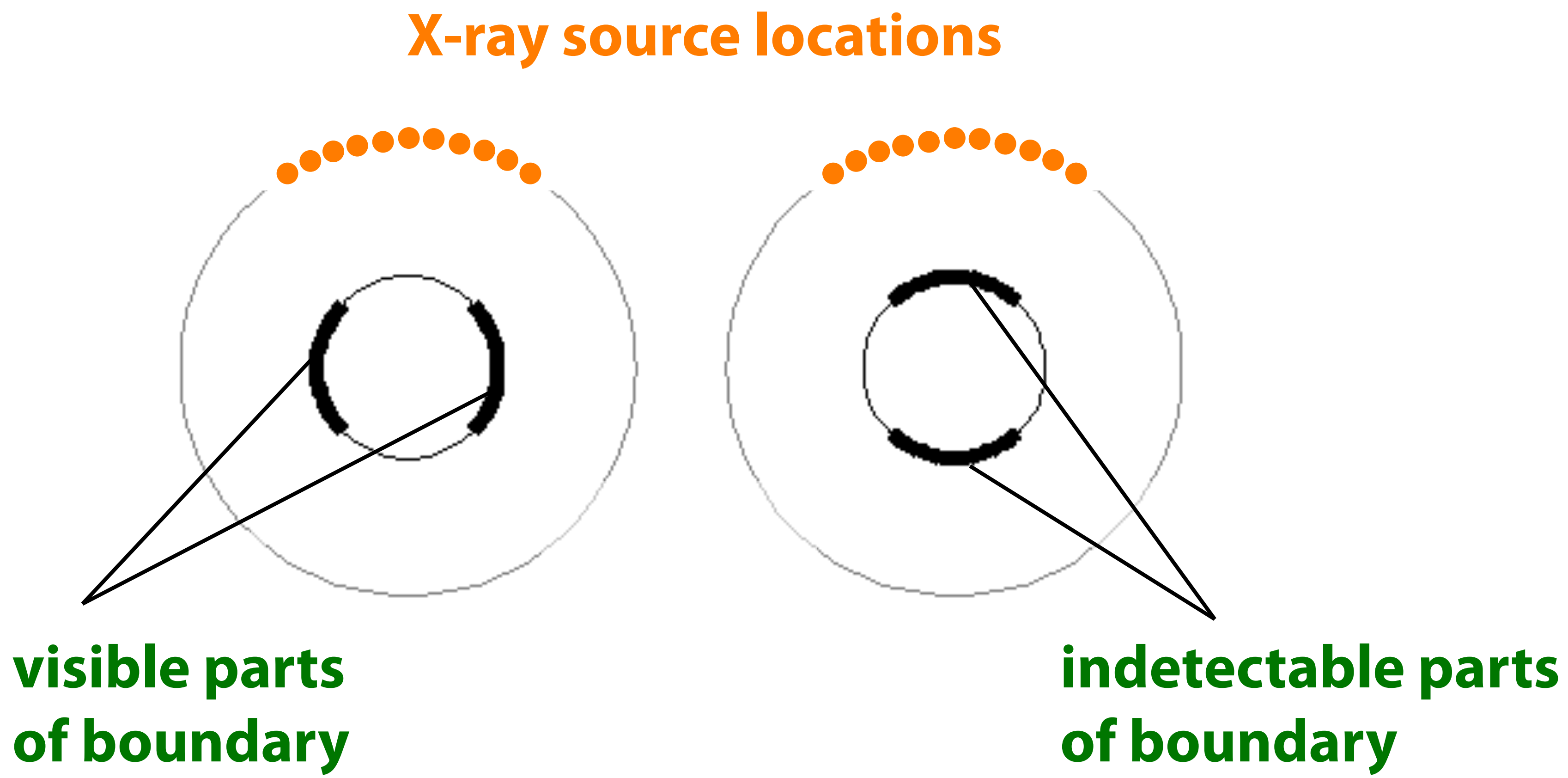

60


90

Horizontal slices:



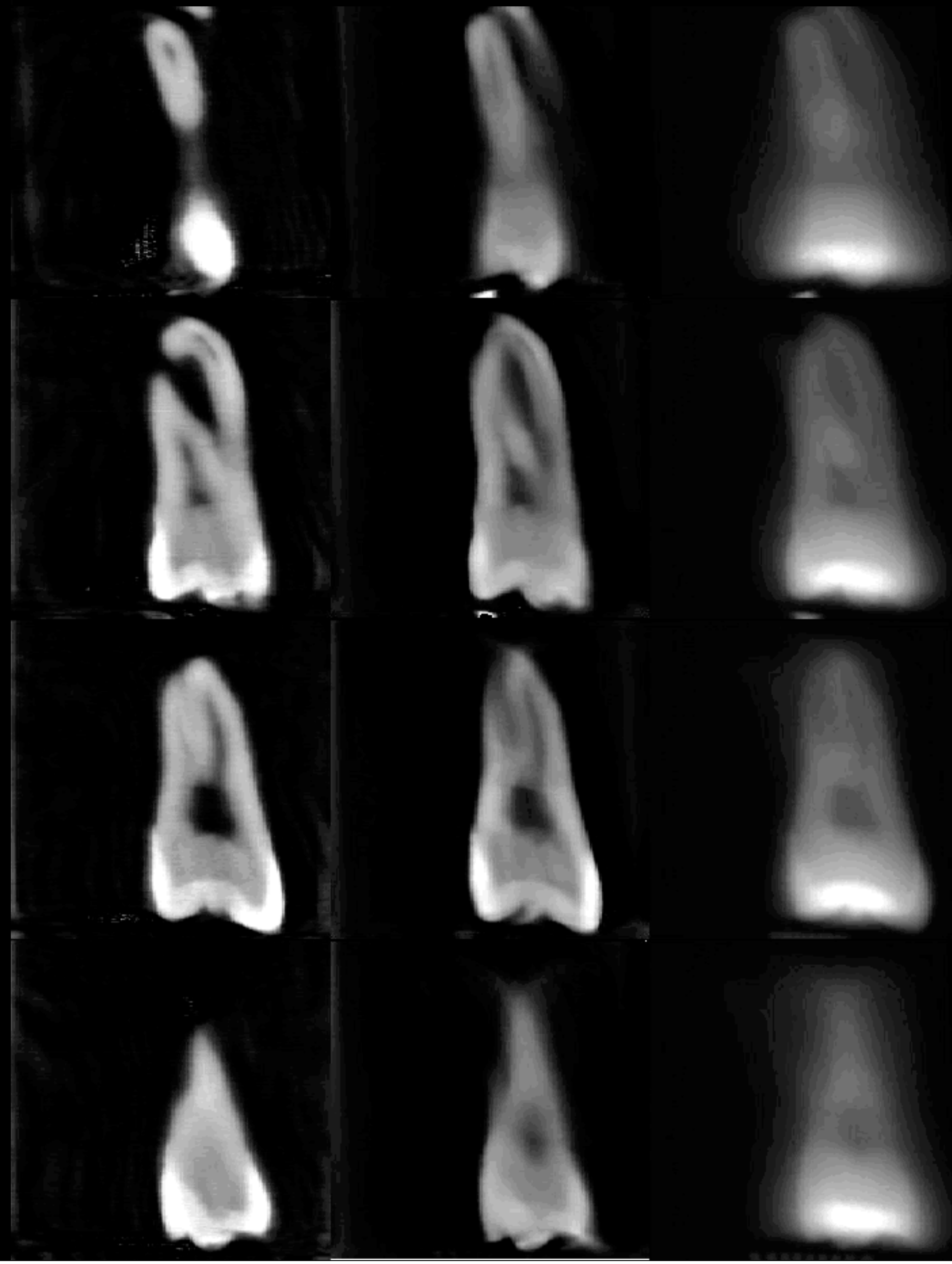
Some parts of the boundary are strongly visible in projection data



Microlocal analysis of recoverable singularities is available in **Quinto** (1993) and **Ramm** & **Katsevich** (1996)

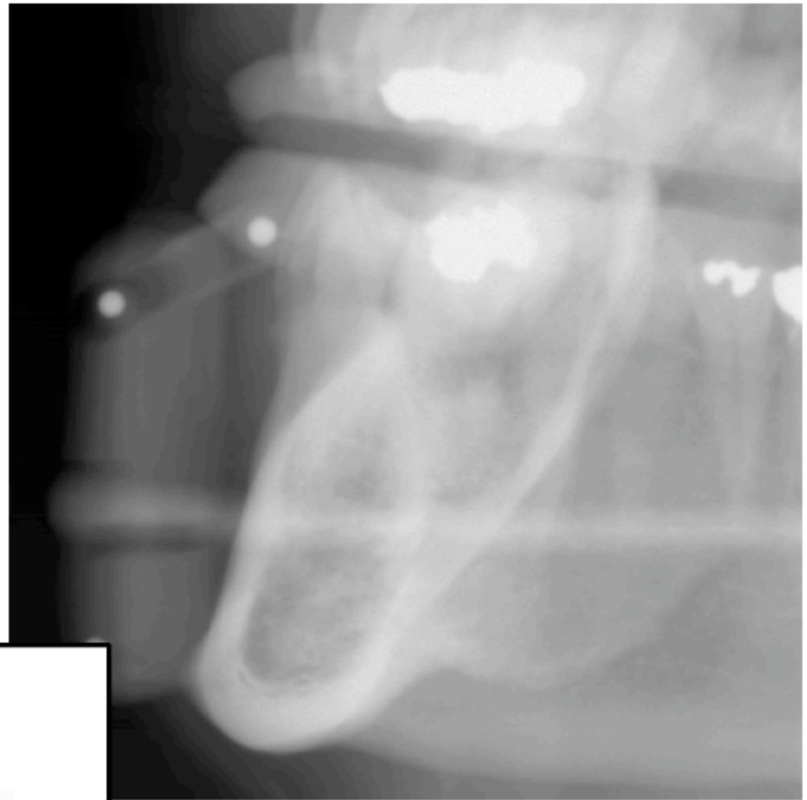
Vertical slices:

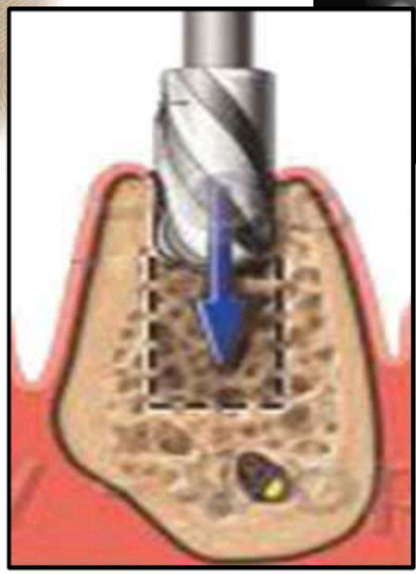
truth full angle **Bayes** limited angle **tomo** limited angle



Kolehmainen, S, Järvenpää, Kaipio, Koistinen, Lassas, Pirttilä, Somersalo (2003)

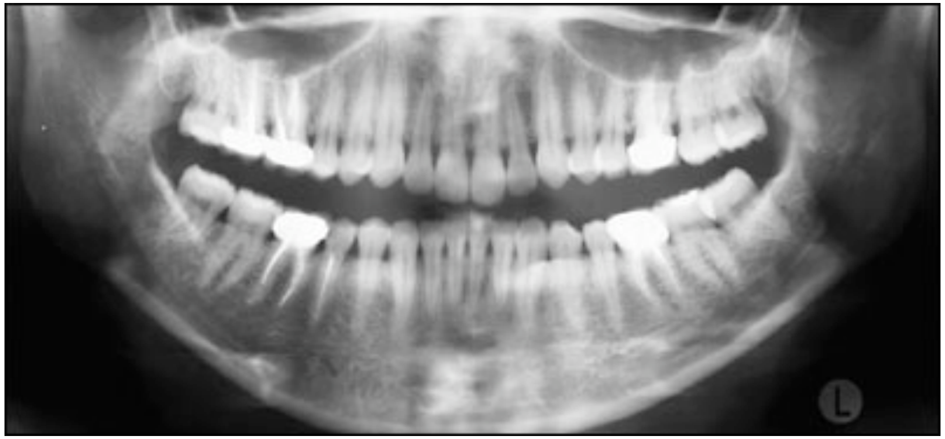
2D projection radiograph is not enough for dental implant planning





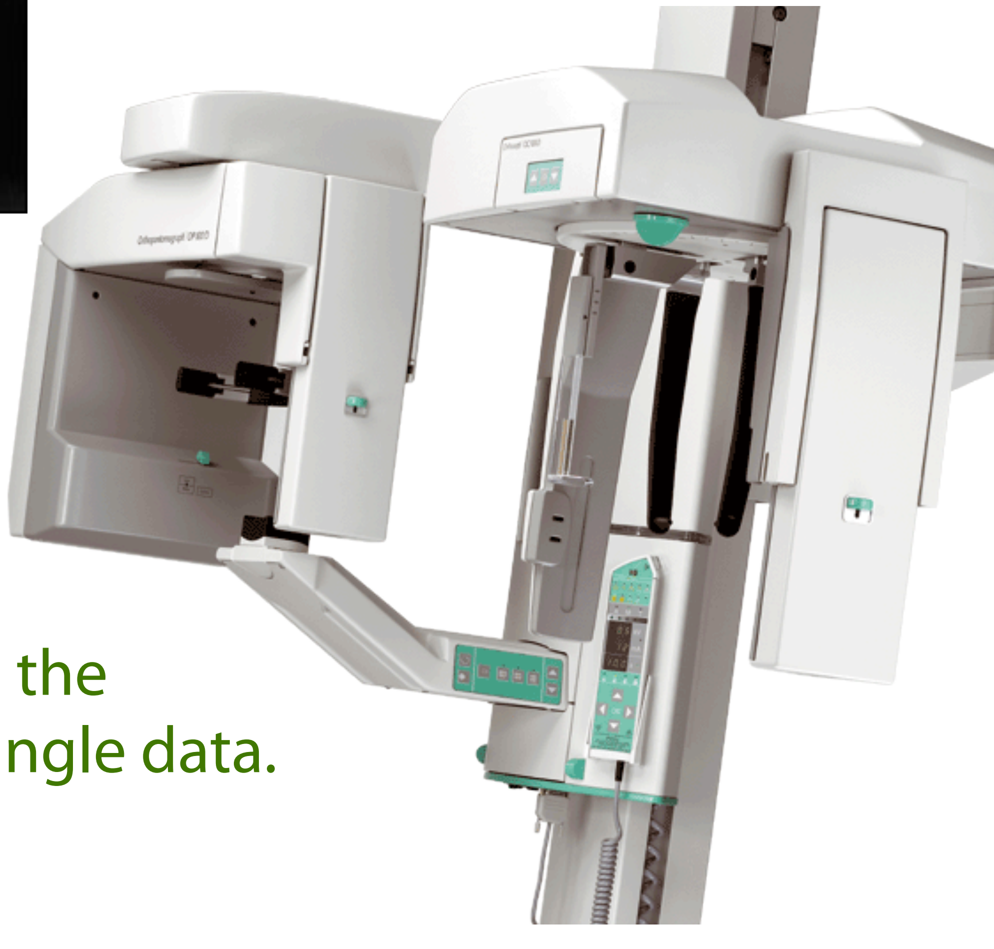
Panoramic X-ray device rotates around the head and produces a general picture

Panoramic imaging was invented by Yrjö Paatero in 1950's.



Nowadays a panoramic device is standard equipment at every dental clinic around the world.

In our project, we reprogrammed the device so that it collects limited-angle data.

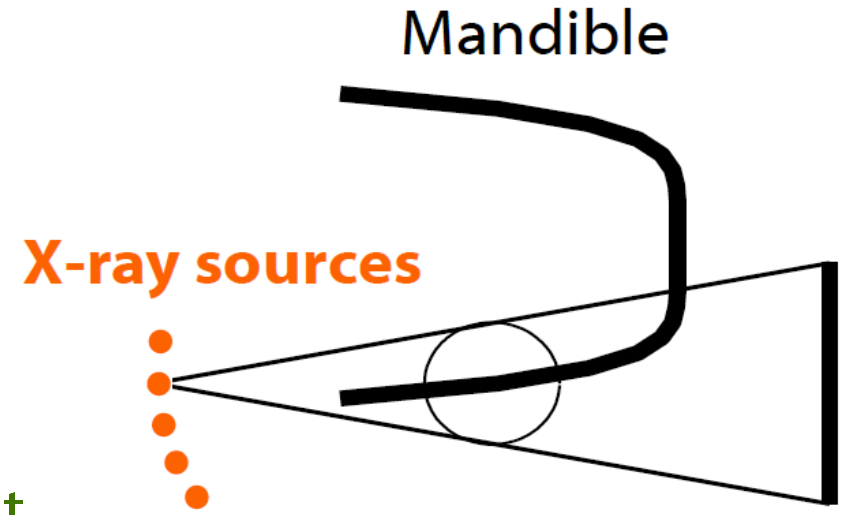


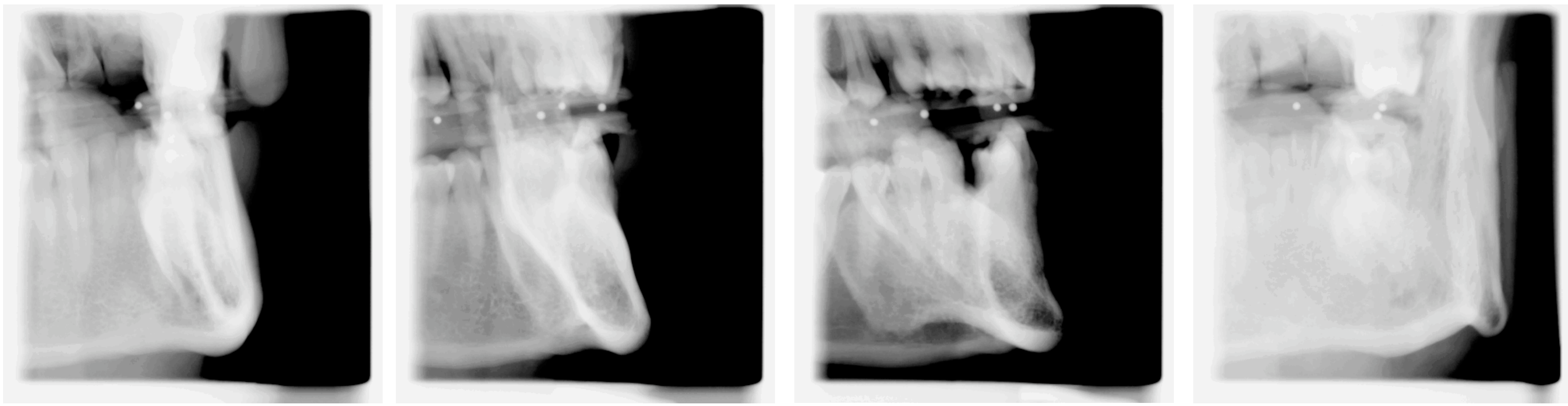
We consider the following limited angle experiment with the panoramic x-ray device:

11 projection images of the mandibular area

40 degrees aperture

1000 x 1000 pixels per image formed by a scanning movement





Limited angle reconstruction can be used for locating the mandibular nerve

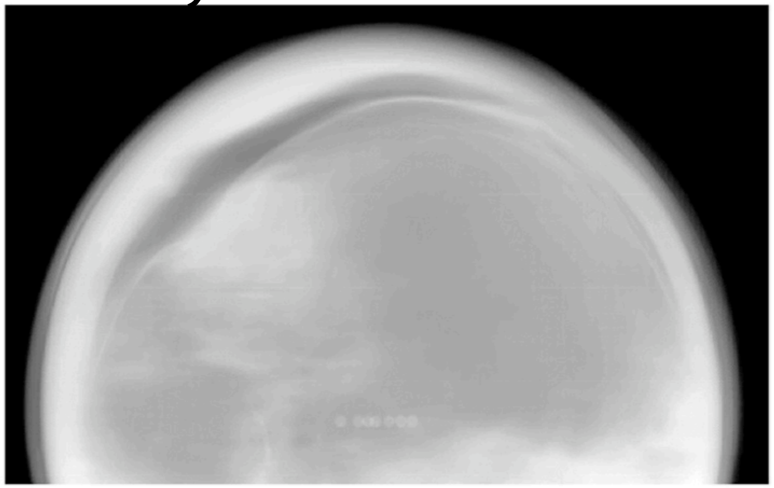
This is core technology for the VT product of PaloDEx Group

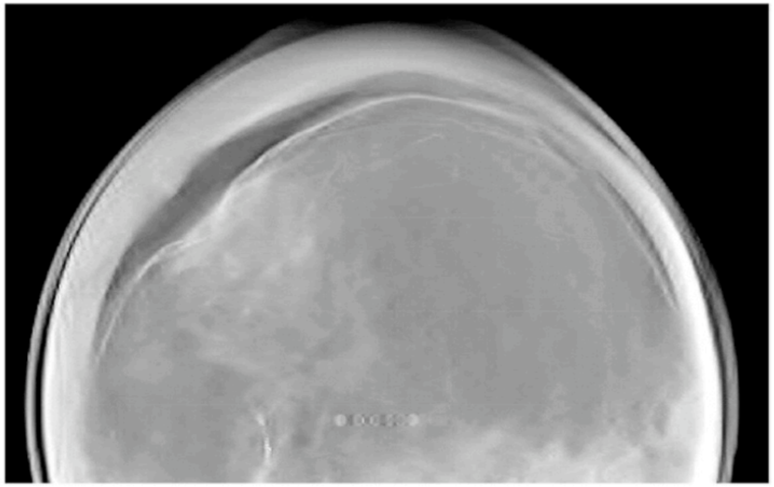
Kolehmainen, Vanne, S, Järvenpää, Kaipio, Lassas and Kalke (2006)
Kolehmainen, Lassas and S (2008)
Cederlund, Kalke and Welander (2009)
Hyvönen, Kalke, Lassas, Setälä, Siltanen (submitted)

Limited angle tomography results for X-ray mammography

[Rantala *et al.* 2006]
Thanks to GE Healthcare

Tomosynthesis



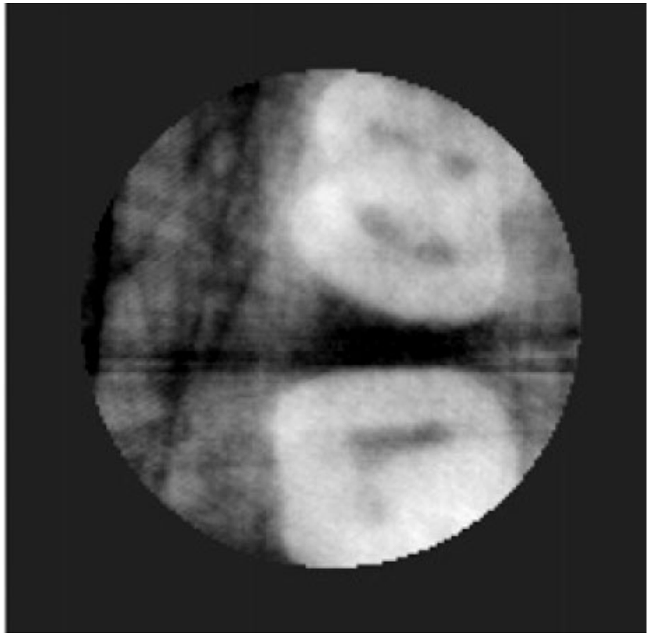


MAP estimate, Besov prior,
p=1.5=q and s=0.5

Local tomography results for dental X-ray imaging; data measured from specimen

Lambda-tomography

MAP using Besov prior with $p=q=1.5$ and $s=0.5$



Niinimäki, S and Kolehmainen (2007)

Thanks to Palodex Group