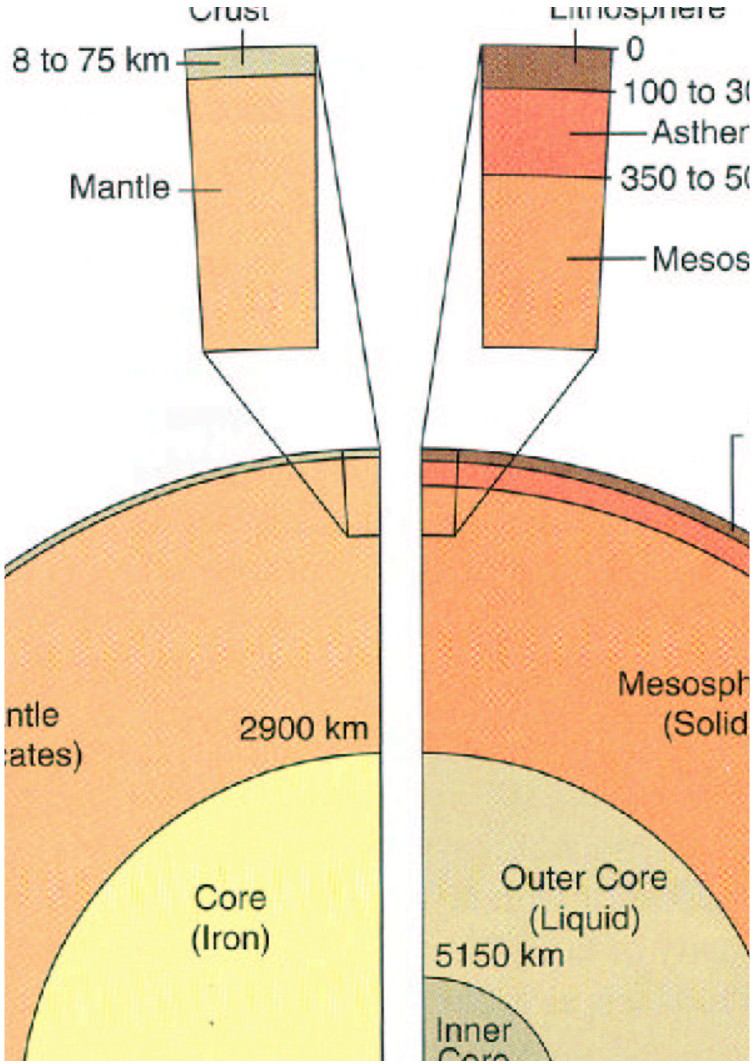
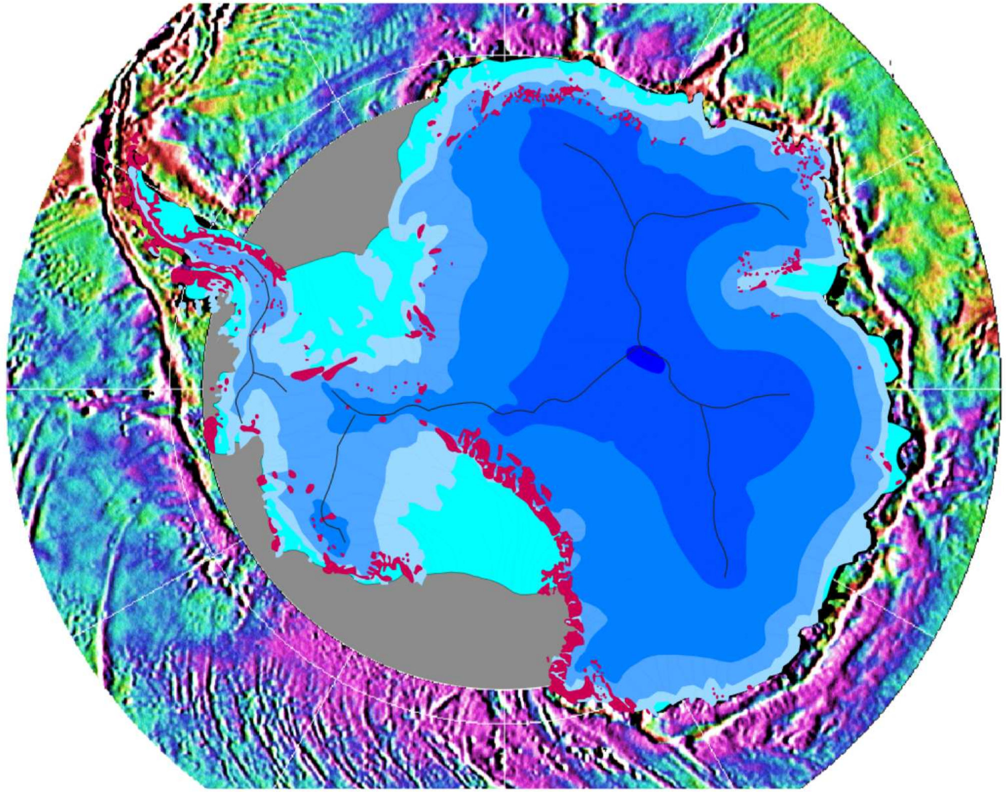
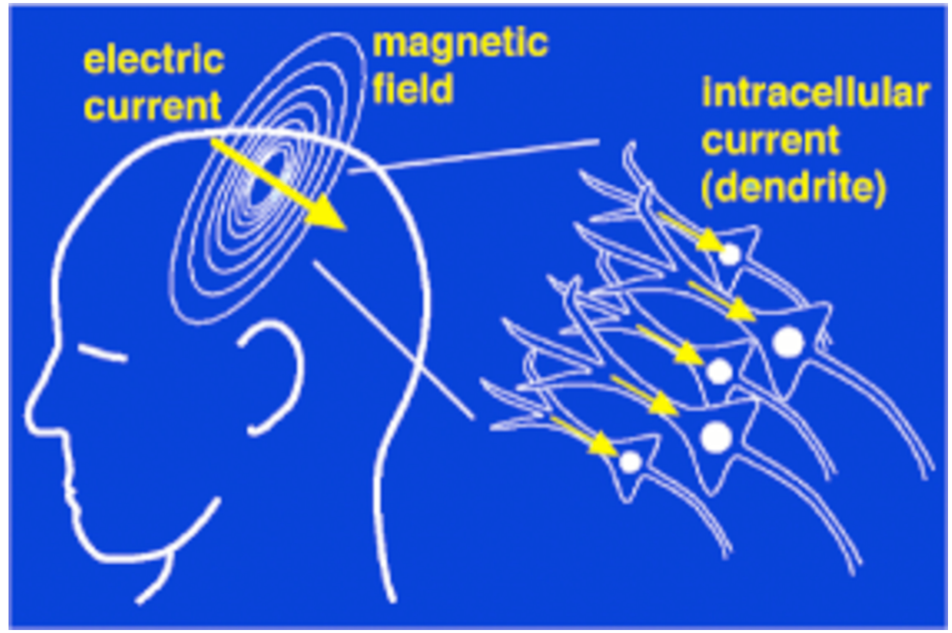
Sovellus: Maanjäristyksillä maan rakennetta selvittämässä:





Painovoiman vaihtelu Antarktiksen lähellä (Kuva: National Oceanic and Atmospheric Administration)



Aivojen toiminnan sähköinen kuvantaminen
(Kuva: Electa/Neuromag)

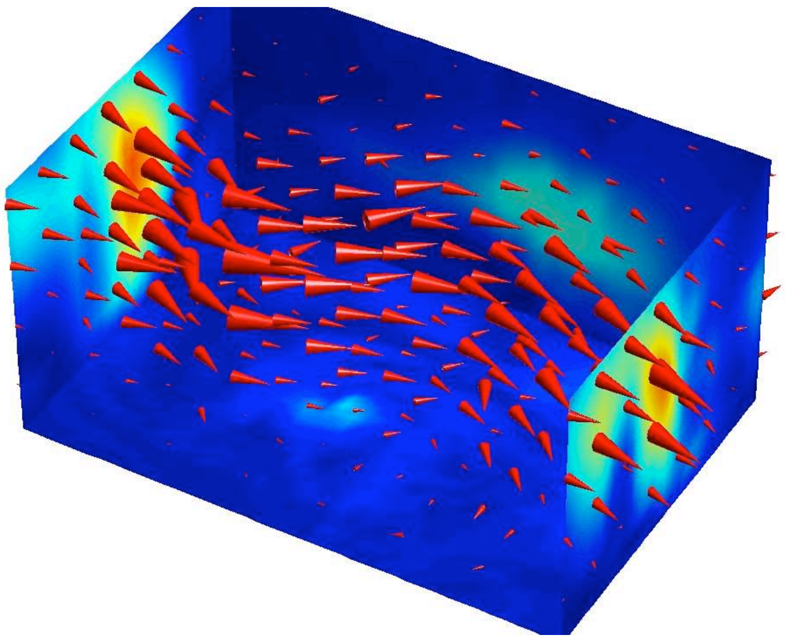
1 Inverse conductivity problem

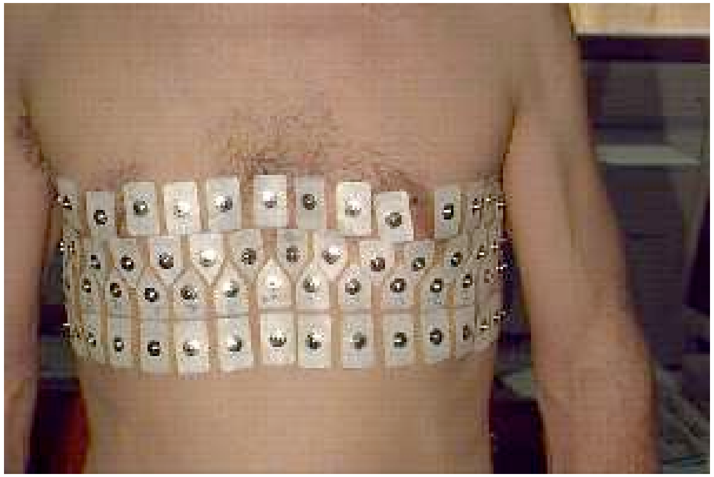
Consider a body $D \subset \mathbb{R}^3$. A potential $u(x)$ causes the current

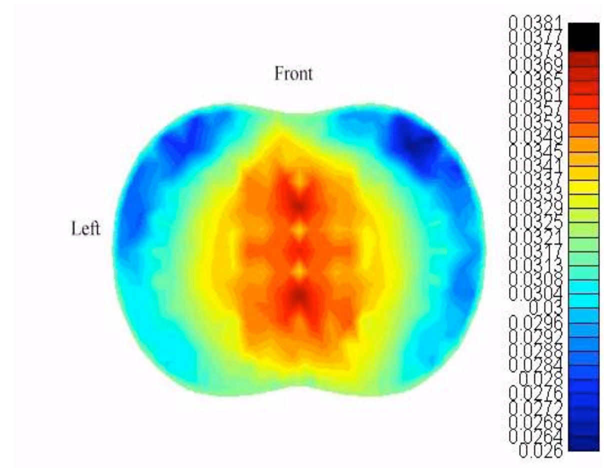
$$J(x) = \sigma(x)\nabla u(x), \quad \text{where } \sigma(x) \text{ is the conductivity.}$$

If the current has no sources inside the body, we have

$$\nabla \cdot \sigma(x)\nabla u(x) = 0.$$







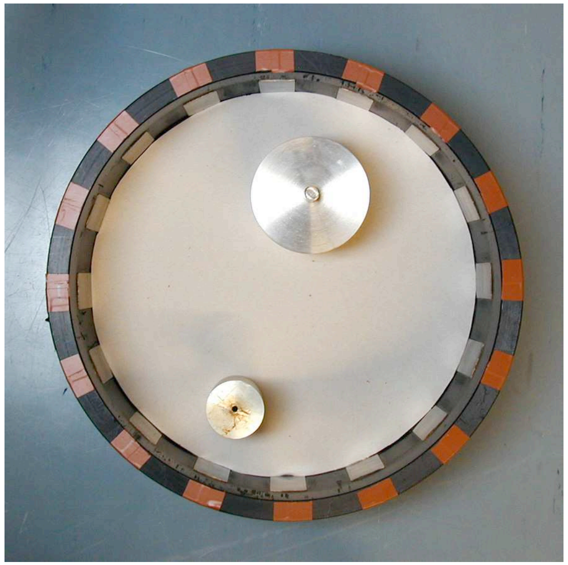
Sähköinen tomografia (Kuva: Oxford EIT lab)

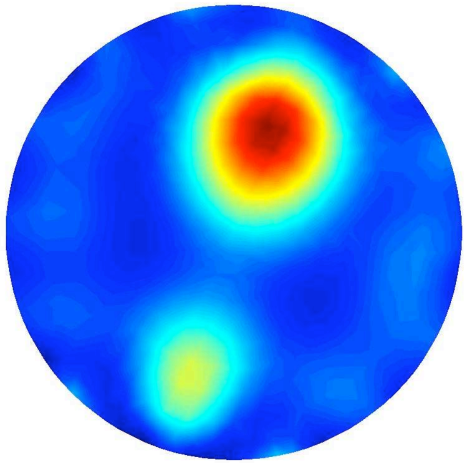
$$\nabla\cdot\sigma(x)\nabla u(x) = 0 \quad \text{in } D,$$
$$n\cdot\sigma\nabla u|_{\partial D} = j.$$

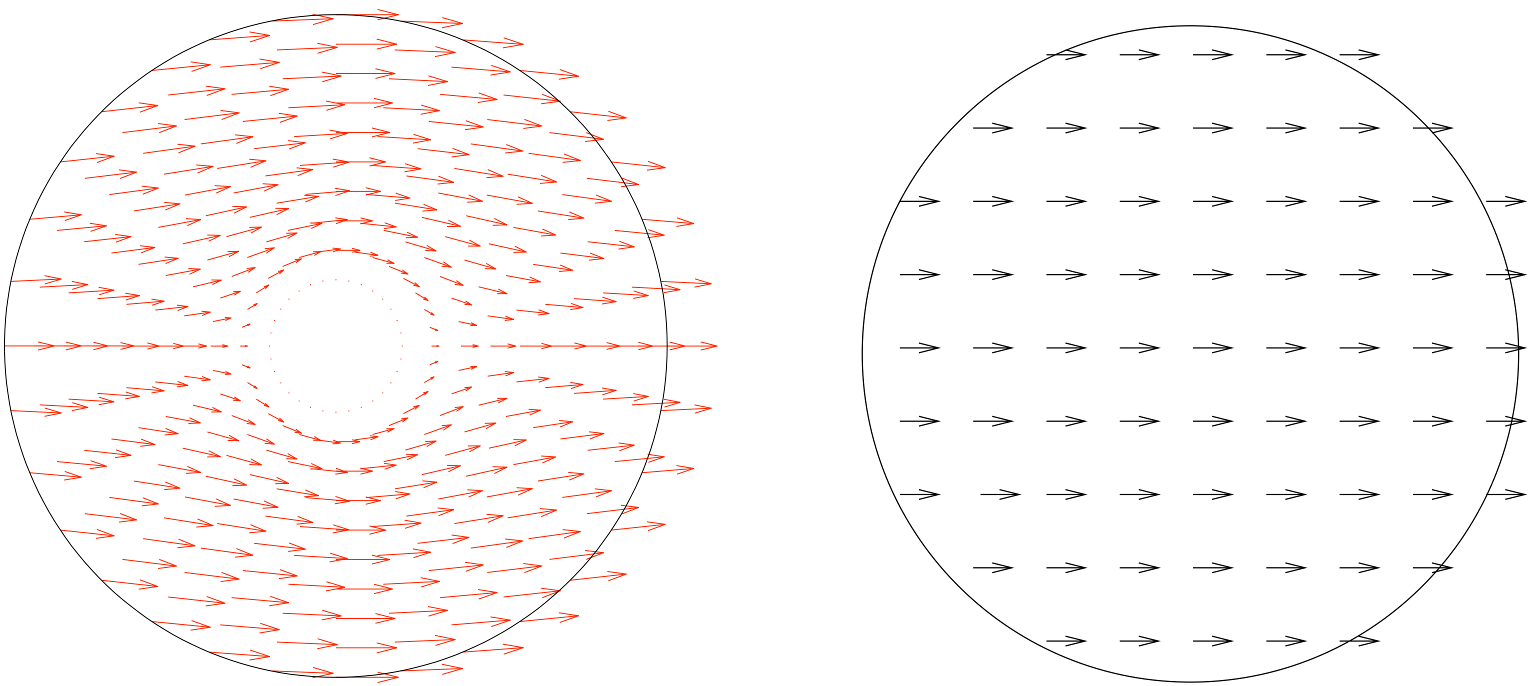
Imaging problem: Do the measurements made on the boundary determine the conductivity, that is, does the map R,

$$R(j) = u|_{\partial D}$$

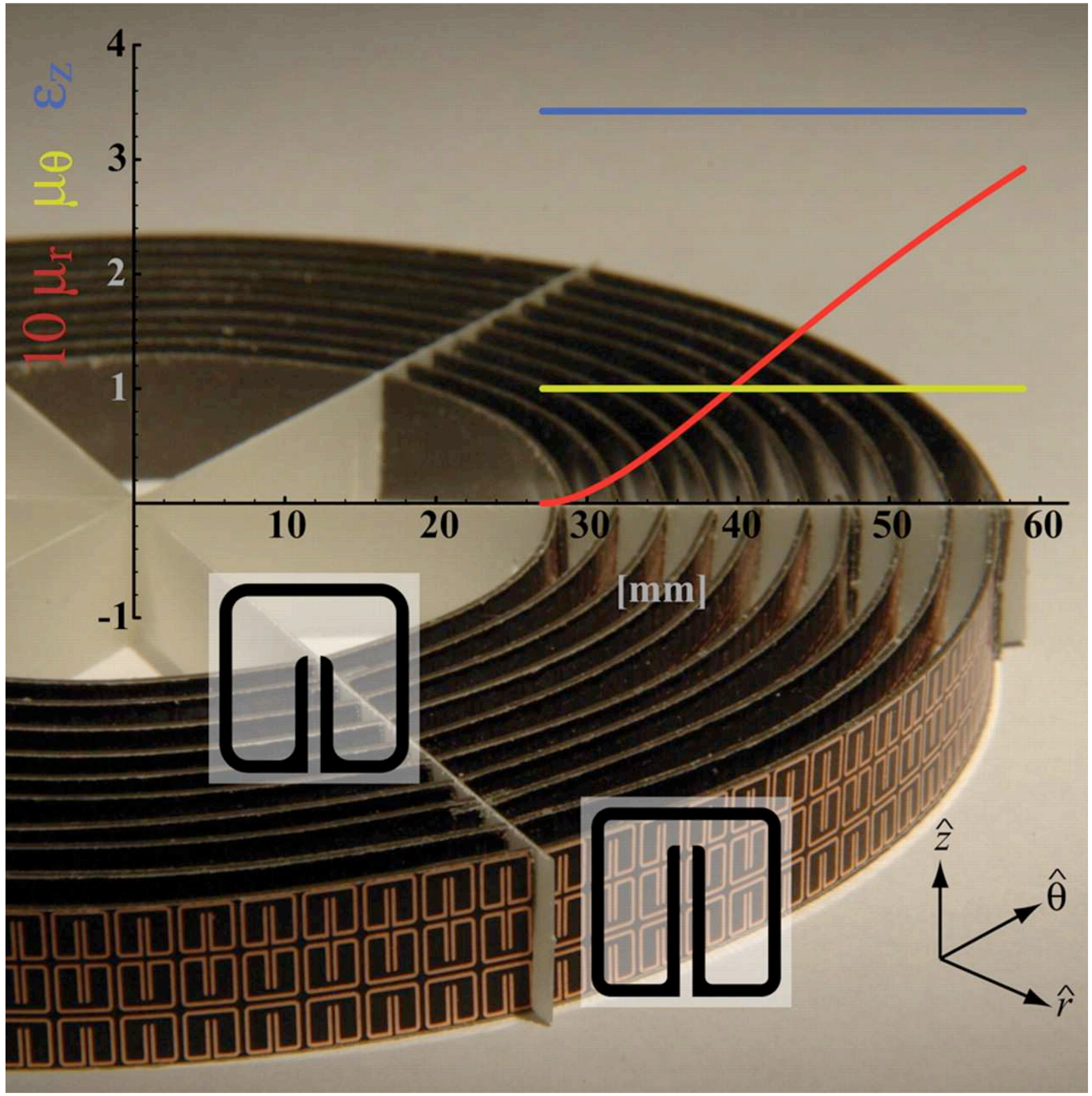
determine the conductivity $\sigma(x)$ in D?



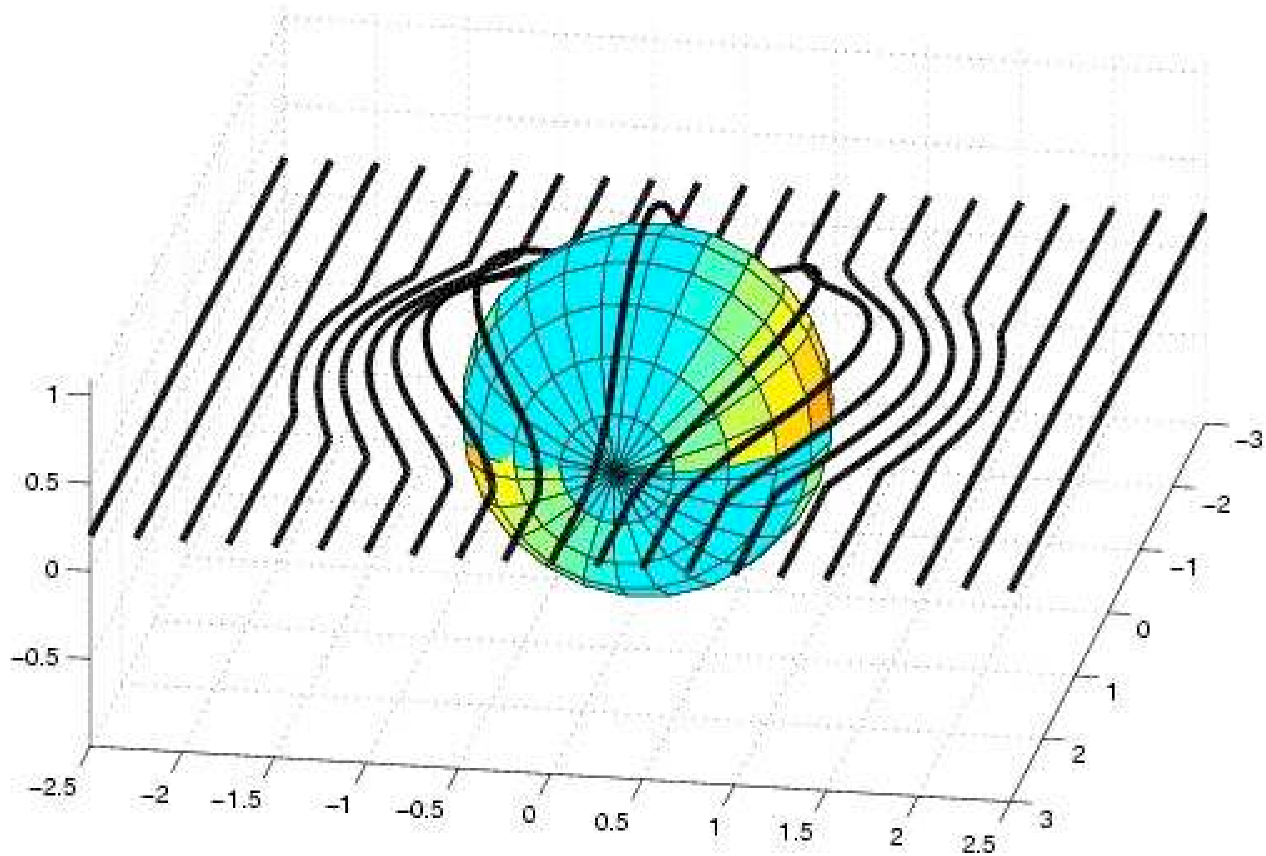




Sähkövirta eksoottisessa aineessa ja tasa-aineisessa kappaleessa



Metamateriaalia mikroaaltotaajuuksille (Schurig et al.)



Valonsäteet kiertävät kappaleen ja palaavat aiemmalle reitilleen

2 Quantum mechanics

Let us continue σ as a constant to $\mathbb{R}^3 \setminus D$. Using the conductivity equation

$$\nabla \cdot \sigma(x) \nabla u(x) = 0 \quad \text{in } \mathbb{R}^3,$$

we define

$$\psi = \sigma^{1/2} u, \quad q(x) = -\frac{\Delta \sigma^{1/2}}{\sigma^{1/2}}.$$

Then ψ is a solution of the Schrödinger equation

$$(\Delta + q(x))\psi = 0 \quad \text{in } \mathbb{R}^3.$$