Exercises 2

1. Show that

$$f(y;\mu,\phi)=\frac{\phi}{\sqrt{2\pi}}\exp\left(-\frac{(y-\mu)^2}{\phi^2}\right),\quad y,\mu\in\mathbb{R},\phi\in\mathbb{R}^+$$

   does NOT belong to the exponential family.

2. Let $y_1,\dots y_n$ be a random sample from $Poisson(\mu)$ where mean value $\mu>0$ is unknown. Show that $\hat{\mu}=\bar{y}$ maximizes the likelihood function

$$L(\mu)=\prod_{i=1}^{n}\left[\frac{\mu^{y_i}}{y_i!}e^{-\mu}\right]$$

3. Let $y_1,\dots y_n$ be a random sample from $Gamma(\beta,\alpha)$ distribution with density function

$$f(y;\beta,\alpha)=\frac{1}{\Gamma(\alpha)}\beta^{\alpha}y^{\alpha-1}e^{-\beta y}.$$

   Derive the formulas for score function, expected information matrix and observed information matrix.

4. Suppose that $Y_1,\dots,Y_n$ are independent and satisfy the linear model

$$\mu_i=\mathrm{E}(Y_i)=\sum_{j=1}^{p}x_{ij}\beta_j$$

   for given covariates x_{ij} and unknown parameters β.

   a) Show that if Y_i has the Laplace distribution

$$f_{Y_i}(y_i;\mu_i,\sigma)=\frac{1}{2\sigma}\exp\left(-|y_i-\mu_i|/\sigma\right)$$

   then the maximum likelihood estimate of β is obtained by minimizing the L_1

$$S_1(y,\hat{y})=\sum|y_i-\hat{y}_i|$$

   over values of $\hat{y}$ satisfying the linear model.

b) Show that if Y_i is uniformly distributed over the range $(\mu_i - \sigma, \mu_i + \sigma)$, maximum-likelihood estimates are obtained by minimizing the L_∞-norm

$$S_\infty(y, \hat{y}) = \max_i |y_i - \hat{y}_i|.$$

5. Show that when the canonical link is used in a generalized linear model, the expected information and observed information are equal.