Bayesian analysis of space shuttle Challenger data

The space shuttle Challenger and its crew of seven people was destroyed soon after its launch on Jan 28, 1986. The disaster was due to leakage of gas from one of the fuel tanks. This was caused by damage to one of the six field-joint O-rings, which are insulating rings made of rubber. At the time of the disaster the temperature was 31 degrees Fahrenheit, which was unusually low. There is data available on the number of field-joint O-rings showing signs of damage and the launch temperature on 23 previous flights.

We analyze the Challenger data using a logistic regression model, which assumes that conditionally on the parameters, the number of damaged O-rings is binomially distributed with probability which depends on the launch temperature. That is,

$$\begin{aligned}[Y_i \mid \beta_0, \beta_1] &\overset{\text{ind}}{\sim} \mathrm{Bin}(6, \pi_i), \\ \mathrm{logit}(\pi_i) &= \beta_0 + \beta_1 x_i, \qquad i = 1, \dots, n,\end{aligned}$$

where $n = 23$ and y_i is the number of damaged O-rings and x_i is the launch temperature on the ith flight. Let Y^* be the number of damaged O-rings at the launch temperature $x^* = 31$. We assume that $Y = (Y_1, \dots, Y_n)$ and Y^* are conditionally independent given (β_0, β_1) and that $p(y^* \mid \beta_0, \beta_1)$ is the binomial distribution with sample size 6 and "success" probability $\mathrm{logit}^{-1}(\beta_0 + \beta_1 x^*)$.

Since formulating the prior directly for the coefficients (β_0, β_1) is difficult, we instead formulate a prior for the parameters

$$\alpha_1 = \mathrm{logit}^{-1}(\beta_0 + \beta_1\, 60), \qquad \alpha_2 = \mathrm{logit}^{-1}(\beta_0 + \beta_1\, 80).$$

We take α_1 and α_2 to be independent and uniform $\mathrm{Uni}(0, 1)$. Since (β_0, β_1) can be expressed uniquely in terms of (α_1, α_2), this implies a prior also for (β_0, β_1).

- Solve (β_0, β_1) in terms of (α_1, α_2).
- Produce a contour plot of the prior in (β_0, β_1)-plane, and add points simulated from the prior on top of the contour plot.
- Write a MCMC algorithm (e.g., independent Metropolis–Hastings with proposals coming from the prior; or random-walk Metropolis). Draw a contour plot of the posterior in (β_0, β_1)-plane, and add points simulated from the posterior on top of the contour plot.
- Plot the predictive distribution $y^* \mapsto p(y^* \mid y)$, where $y^* = 0, 1, \dots, 6$.