Kombinatoriikka (Combinatorics)
Excercise 6

1. Let $n \geq 4$. Count how many permutations $(a_1, \cdots, a_n)$ of the set $[n]$ satisfy the following condition:

a) Then number 1 appears before the number 2 and number 3 appears before the number 4.

b) For each $i, j \in [n]$ the identity $a_i - a_j = a_{n+1-j} - a_{n+1-i}$ holds.

c) Both conditions a) and b).

2. Show using combinatorial reasoning:

a)

$$3^n = \sum_{k=0}^{n} \binom{n}{k} k^2.$$

b)

$$n! = \sum_{k=0}^{n} \binom{n}{k} D_k,$$

where D_k is the number of fixed-point free permutations of the set $[k]$.

3. Let $1 \leq k \leq n$. How many functions $f : [k] \to [n]$ are strictly increasing i.e.

$$f(1) < f(2) < \cdots < f(k)?$$

4. Let $n \geq 1$. On one side of a straight road lie the houses $t_1, \ldots, t_n$ and on the other side houses $h_1, \ldots, h_n$. In how many ways can the houses be painted with k colours available so that neighbouring houses t_i, t_{i+1} or h_i, h_{i+1}, $i \in [n-1]$ get different colours and also houses opposite one another t_i, h_i, $i \in [n]$ get different colours?

5. An event is attended by $l \geq 1$ children and $a \geq 0$ adults. A total of $n \geq l$ (different) prizes are distributed. In how many ways can the prizes be distributed amongst the attendees so that each child gets at least one prize?

The examination takes place at 26.8.2010, 10.00-12.00 in auditorium B123. Students are allowed writing equipment and a basic calculator.