Kombinatoriikka (Combinatorics)
Excercise 1

1. Show that in the 2010 football World Cup tournament the number of teams playing an odd number of matches was even.

[Hint: exactly two teams participated in each match. If $|S|$ is an odd number and for each $s \in S$ there is an odd number $f(s)$ then the sum $\sum_{s \in S} f(s)$ is odd]

2. In how many ways

a) can 10 people be divided into groups of 4 and 6?

b) can 10 people be split into two teams of 5?

3. How many permutations of the set $[8] = \{1, 2, \ldots, 8\}$ map all even numbers to odd numbers?

4. How many strings of length 7 can one form out of the numbers $1 - 7$ such that no odd number appears before a smaller odd number?

5. Let X be a finite set and $f : X \to X$ a mapping. Let us define the sequence $(a_0, a_1, \ldots)$ of numbers as follows :

$$a_k = \begin{cases} |X|, & \text{when } k = 0 \\ |f^k X|, & \text{when } k \geq 1. \end{cases}$$

Show that for all $k \geq 0$ one has

a) $a_k \geq a_{k+1}$

b) $a_k - a_{k+1} \geq a_{k+1} - a_{k+2}$