The D-bar method for nonlinear inverse problems

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The D-bar method for inverse problems originates from the pionering work of Gardner, Kruskal et. al. in the late 1960's on the construction of soliton solutions for the KdV-equation. This construction is based on a non-linear Fourier transform, a so-called scattering transform and associated inverse scattering transform, which makes the time-evolution linear. Therefore the method is called the inverse scattering method. It is also known as the D-bar method, since a crucial step in the method involves taking the derivative with respect to the complex conjugate of a certain parameter. In the 1980's the D-bar method was realized as a solution method to certain non-linear inverse scattering and inverse boundary value problems [BC89]. The method gives rise to reconstruction algorithms, which has during recent years been shown to be numerically efficient [KMS04].

In this series of lectures we will introduce the D-bar method in the context of Electrical Impedance Tomography (EIT) and discuss the relationship to certain non-linear evolution equations. We will outline a reconstruction method for the full non-linear problem in EIT based on the D-bar method, and show how the method is implemented numerically.

References

[BC89]   Richard Beals and Ronald R. Coifman. Linear spectral problems, non-linear equations and the $\bar{\partial}$-method. *Inverse Problems*, 5(2):87–130, 1989.

[KMS04] Kim Knudsen, Jennifer Mueller, and Samuli Siltanen. Numerical solution method of the dbar-equation in the plane. *Journal of Computational Physics*, 198(2):500–517, 2004.

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