Elementary Bayesian Analysis **example exam questions**

Use equations to clarify your answers if necessary.

1. Describe briefly (total of about one page) the following terms and what is their significance in Bayesian analysis: (in a real exam there is 6 terms asked, and below is collection of terms from previous exams)
   - Marginal distribution (1p)
   - Posterior distribution (1p)
   - Predictive distribution (1p)
   - Posterior predictive distribution (1p)
   - Semi-conjugate prior (1p)
   - Non-informative prior distribution (1p)
   - Improper prior (1p)
   - Improper distribution (1p)
   - HPD-interval (1p)
   - Observed information (1p)
   - Under-identified model (1p)
   - Non-identifiable parameter (1p)
   - Aliasing (1p)
   - Hyper-parameter (1p)
   - Monte carlo error (1p)
   - Grid sampling (1p)
   - Inverse-cdf -method (1p)
   - Convergence (1p)
   - Burn-in (1p)
   - Sensitivity analysis (1p)
   - Bayes factor (1p)
   - Utility function (1p)
   - Ignorability (1p)
   - Stable treatment (1p)
2. Compare pros and cons of Gibbs sampling and Metropolis-Hasting algorithm and describe how the choice of MCMC-algorithm may be affected by the model. (6p)
3. Describe what is convergence diagnostics and why it is needed. Describe one approach with justification, pros and cons. (6p)

4. Markov chain Monte Carlo (MCMC)

   a) Describe briefly Gibbs sampling and Metropolis-Hasting algorithms and compare pros and cons. (3p)

   b) Describe what is convergence diagnostics and why it is needed. Describe one approach briefly with pros and cons. (3)

5. Describe most used methods to summarize posterior and predictive distributions. Mention also the possible problems of these methods. (6p)

6. Markov chain Monte Carlo (MCMC)

   a) How one can estimate how many MCMC samples are needed? (3p)

   b) Describe briefly Metropolis-Hasting algorithm. What issues have an effect to its performance and how the performance can be improved? (3p)

7. Markov chain Monte Carlo (MCMC)

   b) Describe briefly Gibbs sampling and mention at least one good and one bad property (3p)

   b) Describe briefly PSRF diagnostics and mention at least one good and one bad property (3p)

8. Markov chain Monte Carlo (MCMC)

   a) Describe briefly Metropolis algorithm and mention also its good and bad properties. (3p)

   b) What is the transition distribution produced by Metropolis algorithm? (1p)

   c) Describe the relation between Gibbs and Metropolis algorithms. (2p)

9. Explain Monte Carlo error, how it can be estimated in different situtations, and give an simple example. (6 p)

10. Write about one page essay on Bayesian model checking, evaluation and comparison. (6p)

11. Briefly discuss Bayesian model checking and evaluation. (6p)

12. Describe posterior predictive checking and its connection to external validation. Mention also at least one good and bad property. (6p)

13. Discuss Bayesian model checking and evaluation. (6p)

14. Describe posterior predictive checking and its pros and cons. (6p)

15. Model checking and comparison

    a) Describe DIC and mention at least one good and one bad property (3p)

    b) Describe posterior predictive checking and mention at least one good and one bad property (3p)

16. Describe posterior predictive checking and mention at least one good and one bad property. (3p).

17. Describe sample survey, designed experiment and observational study and main differences between them (6p)

18. Explain why exchangeability is important concept in bayesian modeling. (3p)

19. Exchangeability
    a) Explain briefly the term exchangeability. (3p)
    b) Does exchangeability hold in the following examples? Justify. (3p)
    i) Exam results for ten students $T_1, \ldots, T_{10}$.
    ii) Exam results for ten students $T_1, \ldots, T_{10}$, when we know that exactly half of the students are gradauate students.
    iii) Exam results for ten students $T_1, \ldots, T_{10}$, when we know the average exam results for those students $X_1, \ldots, X_{10}$.

20. Explain terms informative and non-informative prior, and explain their purpose, pros and cons. (6p)

21. Exchangeability
    a) Explain briefly the term exchangeability and give an example. (2p)
    b) Does exchangeability imply independence? Explain. (2p)
    c) How is exchangeability related to hierarchical models? Give an example. (2p)

22. Exchangeability
    a) Explain briefly the term exchangeability. (2p)
    b) Does exchangeability imply independence? Explain. (2p)
    c) What happens to exchangeability if additional information related to units is obtained? (2p)

23. Describe briefly following methods and mention at least one good and bad property for both of them
    a) Normal distribution approximation (3p)
    b) Gibbs sampling (3p)

24. Decision analysis
    a) Describe shortly the Bayesian decision analysis process (principle, terms, and steps) (3 p)
    b) Compute the expected cost in the following example (maximum 2 digits accuracy) (3 p)

    Patient has some symptoms X and goes to see a doctor. Symptoms might be caused by disease Y or Z. Only disease Y can be cured with antibiotics. Time in bed for disease Z is 5 days. Time in bed for disease Y without antibiotics is 7 days. With antibiotics symptoms are relieved in 3 days. With present symptoms prior for disease Y is 5%. Disease Y can be tested with laboratory test, which accuracy is 98%. Receiving the test reult takes one day. If the cost is time (time in bed + optional waiting of the test result), compare costs if 1) no antibiotics, 2) antibiotics right away without test. 3) antibiotics only if the test result is positive? If antibiotics had a cost how many days it would have to correspond so that it would better not to eat antibiotics.

25. Decision analysis
    a) Describe shortly the Bayesian decision analysis process (principle, terms, and steps) (2 p)

b) A part of an aircraft engine can be given a test before installation. The test has only a 75% chance of revealing a defect if it is present, and the same chance of passing a sound part. Whether or not the part has been tested it may undergo an expensive rework operation which is certain to produce a part free from defects. If a defective part is installed in the engine the loss is L. If the rework operation costs L/5 and 1 in 8 of parts are initially defective, calculate how much you could pay for the test and determine all the optimum decisions. (4p)

26. Decision analysis

    a) Describe shortly the Bayesian decision analysis process (principle, terms, and steps) (2 p)

    b) Solve following decision analysis problem (4p)

    Company X has developed a new product. It is now time to decide whether to start production and sales. The success of the product is not certain. The company's estimate is that if the product is success it will produce 10 million euro profit, and if it is disaster it will produce 5 million euro loss. Previous development costs are not considered here, and thus if production of the product is not started, the loss will be 0 euro. The company's market expert estimates that the probability of success is 0.3.

    - Should the company start the production and sales of the product?

    - If market research could tell precisely whether the product is success or disaster, what is the maximum worthwhile price of such perfect information for the company?

    It is possible to produce a smaller trial batch to be sold in a smaller market area. Based on the success of this smaller trial batch, it is possible to infer possible success in the whole market area. The cost of the trial batch would be 1 million euro. If the trial batch is success, the company would get back 1.2 million euro. If the trial batch is disaster, the company would get back 0 euro. Based on previous products it is known, that if the product was success in the whole market area, then it had been success in the smaller market area with 90% probability, and if the previous product had been disaster in the whole market, then it had been disaster in the smaller market with 80% probability.

    - Should the company make smaller trial batch?

    (Note: Here the problem is naturally simplified. In real scenario the estimated profits and related uncertainties would be continuous valued, but the principle of inference would be same.)

27. Decision analysis

    A patient may have a severe disease. Without an operation the probability of recovery is 5%. If the patient with the disease is operated, the probability of recovery is 50%. Operation is dangerous and thus even if the patient does not have the disease there is 20% chance that the patient dies as a result of the operation.

    If thw operation is done and the patient lives is recovery complete. If the operation is not done and the patient has the disease, recovery is only partial. If the patient does not have the disease and the patient does not die is recovery complete.

    Advise the doctor in making the decision to operate. You may use following costs: complete recovery = 0, partial recovery = λ, and death = 1.

28. Describe shortly the Bayesian decision analysis process and compute the expected cost in the following example (maximum 2 digits accuracy). (6p)

In the morning you need to make a decision about what to do. Your choices are
a_1: "stay home, a_2: "go out without umbrella, a_3: "go out with umbrella

The possible states of the world are
$\theta = 0$: "no rain $\theta = 1$: "rain,

Your observation is the weather forecast: no rain ($y = 0$) or rain ($y = 1$).

The costs and distribution of the observations are described in the tables below:

	weather	
action	$\theta = 0$	$\theta = 1$
a_1	4	4
a_2	0	5
a_3	5	2

	weather	
observation	θ_1	θ_2
$y = 0$	0.9	0.2
$y = 1$	0.1	0.8

Your prior is $p(\theta = 0) = p(\theta = 1) = \frac{1}{2}$. The weather forecast predicts rain ($y = 1$). Compute the expected cost for all actions and give the action which minimizes the expected cost.