Decision analysis (chapter 22)

- Steps of Bayesian decision analysis
  - decision or action
  - outcome and its distribution
  - utility or cost function
  - distribution of utility given decision
  - expected utility
  - maximization of expected utility
- Simple decision analysis calculation
  - similar to examples in chapter 22.3 or exercises 22.1

Slide 1

Example – medical decision making

- Observations on population with some acute disease
  - 30-day mortality y_i
  - p measurements $(x_{i1}, \ldots, x_{ip})$ which might help to predict mortality
- Goal is to predict the probability of death for a patient
  - different costs for the measurements
  - limited amount of money
  - estimate the goodness of predictions
  - which measurements should be made?

Slide 2

Example – medical decision making

Slide 3

- Draper & Fouskakis analyse pneumonia
  - 30-day mortality y_i
  - 2500 patients
  - measurement cost consists mainly of personnel costs
  - costs for outcomes given decisions were based on discussion with health care specialists
  - sensitivity analysis for cost estimates was also made
- Result
  - choosing the most useful measurements could save about US$8 per patient

Bayesian decision analysis

Slide 4

- Decisions d
  - sometimes called actions a
- Potential consequences x
  - x is nominal, ordinal, real, scalar, vector, . . .
- Probability distributions of consequences given decisions $p(x|d)$
  - in decision making the decisions are controlled, and thus $p(d)$ does not exist
- Utility function $U(x)$ maps consequence to real value
  - e.g. euro or expected life time
  - sometimes term cost or loss used instead
- Expected utility $E[U(x)|d] = \int U(x)p(x|d)dx$
- Choose decision d^*, which maximizes the expected utility

$$d^* = \arg\max_d E[U(x)|d]$$

Distribution of the utility and the expected utility

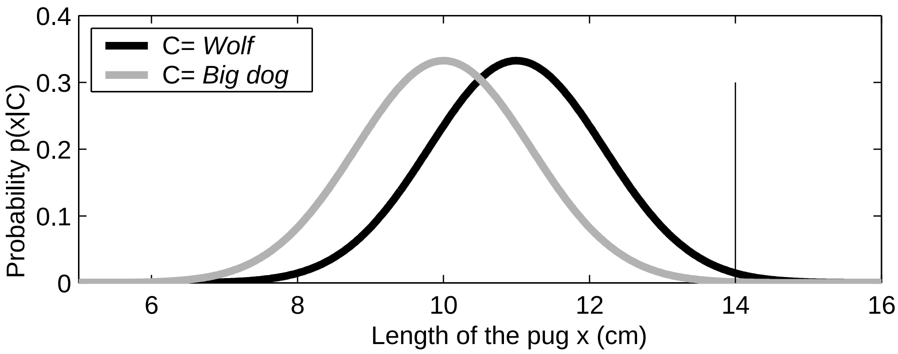
Slide 5

- In theory optimal rational decision is based on the expected utility (expectation of the distribution of the utility)
- The distribution of the utility can be, however, used in sensitivity analysis
- When examining the distribution of the utility, it may be found out that the utility function used was not appropriate
  - Eg: utility is money
    - expectation may be positive, but distribution may indicate chance of very large loss, which might lead to a loss of home, for example
    - if the uncertainty presented by the distribution would affect the decision, the original utility function is imperfect
    - utility function should include the cost of uncertainty and, eg. costs related to losing home

Example: 2 choices

Slide 6

- Peter is going to pick berries in the forest, but on the way notices a pug, which might be made by a dog or a wolf
- Peter measures the pug to be 14 cm and goes home to check his Encyclopedia of animals for statisticians for reference on paw sizes of dogs and wolves



- Based on his encyclopedia, probability of a wolf is 0.92 (compared to a dog)

Example: 2 choices

- Peter also assumes that there are a hundred times more free running dogs, than wolves in the area, ie. *a priori* probability for a wolf is about 1%.
- Likelihoods and posterior probabilities

Slide 7

Class	Likelihood	Posterior probability
Wolf	0.92	0.10
Big dog	0.08	0.90

- Probability of a wolf is 0.10

Example: 2 choices

- Peter considers his options
- If Peter stays home he don't get berries
- If Peter goes to forest and it's a dog pug he gets berries
- If Peter goes to forest and it's a wolf pug, he may get eaten by the wolf

Slide 8

	Class	
Action d	Wolf	Dog
Stay home	1	1
Go pick berries	1000	0

Loss $U(x)$

	Conditional loss
Action d	$E[U(x)\|d]$
Stay home	1
Go pick berries	100

Losses for actions

Example: 2 choices

- Maximum likelihood classification is a wolf
- Maximum a posteriori classification is a dog
- Minimum loss decision is to stay home
  - cost of being eaten by wolf is big
- Example illustrates that also events with low probability need to be included in the final decision making if they have large costs

Slide 9

Example: many choices

- Prof. Gelman has a jar of quarter dollars
  - first a line is drawn on the jar, and the jar is filled with coins up to the line, and thus number of coins in the jar is unknown
  - Prof. Gelman does not himself know the number of coins
  - Prof. Gelman offers the class the chance to win the all coins in the jar, if the class guesses the number of coins correctly
- Esim9_1.m

Slide 10

Expected utility and utility function for money?

Slide 11

- Utility function for money is not usually linear
  for example:
  - certainly 0€ or 50% chance of getting a million and 50% chance of losing a million (not many have the possibility to pay the loss)
  - certainly 0€ or 50% chance of getting 1.1 million and 50% chance of losing a million (expected utility is positive, bu the risk is big)
  - pay certainly 0.48€ or play one line of lotto
    (over ten million lines of lotto played in Finland every week)
  - insurance vs. no insurance
    (expected utility of insurance is negative)

Difficulties in selection utility function

Slide 12

- People are not good at estimating probabilities, especially very small probabilities
- Fear of uncertainty ie. risk aversion is difficult to formalize
- Cost of uncertainty is difficult to formalize
- It is difficult to estimate benefits and costs eg. in health care
  - what diseases should be treated and with how large cost?
  - utility for a person is her and her close ones good health
  - utility for a doctor might include bonus for how saving money
- What is the cost of polluted environment and global warming
- Often final decision making is politics

Multistage decision making (ch 22.3)

- 95 year old with a tumor, which is malignant with 90% probability
- Based on statistics
  - life expectancy is 34.8mo if no cancer
  - life expectancy is 16.7mo if cancer and radiotherapy
  - there is 35% chance of death in surgery, but if survives, life expectancy is 20.3mo
  - life expectancy is 5.6mo if cancer and no treatment
- Which treatment should be chosen?

Slide 13

Multistage decision making

- Which treatment should be chosen?
  - estimate quality-adjusted life expectancy
  - subtract one month due to treatment-caused discomfort
- Quality-adjusted life expectancy
  - Radiotherapy: 0.9*16.7 + 0.1*34.8 .1 = 17.5mo
  - Surgery: 0.35*0+0.65*(0.9*20.3+0.1*34.8-1) = 13.5mo
  - No treatment: 0.9*5.6 + 0.1*34.8 = 8.5mo

Slide 14

Multistage decision making

- Optional test, bronchoscopy
  - 70% pr. of detecting the cancer, if malignant
  - 2% pr. of detecting the cancer, if non-malignant
  - 5% pr. that patient dies due to complications
- Is it worth making the additional test?
  - Bayes rule is used to update probabilities
  - in this case, using updated probabilities, decision would be same, and bronchoscopy should not be made

Slide 15

Price of an additional life month

- What if the price of the treatments would be included?
  - price for an additional life month for 95-year-old
  - price for an additional life month for 45-year-old
  - price for an additional life month for 5-year-old
- Decision analysis can help answering these, but final answer is politics

Slide 16

Decision making in design of experiment

- What kind of experiment would provide maximum information
  - choose optimal value for x_{n+1}
- Ex 22.2
  - assume that in bioassay posterior accuracy is not enough to decide toxicity class
  - choose optimal dose, which reduces the posterior variance of LD50 as much as possible?
    - this way less animals needed
  - to simplify, in exercise, you may use same dose levels as in book

Slide 17

Decision making in design of experiment

- Ex 22.2
  - for each dose compute utilities for consequences, i.e. how much posterior variance changes if
    - animal dies
    - animal does not die
  - for each dose compute the probabilities of consequences
    - animal dies
    - animal does not die
  - combine these to get expected utility (expected reduction in posterior variance)

Slide 18