Exercises 6

1. Suppose that Y_1 and Y_2 are independent Poisson random variables with means μ and $\rho\mu$, respectively. Using the result

$$Y_1 + Y_2 \sim \text{Poisson}(\mu + \rho\mu)$$
$$Y_1 \mid Y_1 + Y_2 = m \sim \text{Bin}(m, 1/(1 + \rho))$$

   construct a test for the $H_0 : \rho = 1$ against the one-sided alternative $H_1 : \rho > 1$.

2. Consider the dose-response model

$$g(\mu) = \beta_0 + \beta_1 x.$$

   Under the hypothesis that the response probability at x_0 is equal to μ_0, show that the model reduces to

$$g(\mu) = \beta_0(1 - x/x_0) + g(\pi_0)x/x_0.$$

   How would you fit this model using R?

3. Logistic discrimination: Suppose that a population of individuals is partitioned into two sub-populations or groups, G_1 and G_2. Measurements $\mathbf{Z}$ made on individuals have the following distributions in the two groups:

$$G_1 : \quad \mathbf{Z} \sim N_p(\mu_1, \boldsymbol{\Sigma})$$
$$G_2 : \quad \mathbf{Z} \sim N_p(\mu_2, \boldsymbol{\Sigma}).$$

   Let $\mathbf{z}^*$ be an observation made on an individual drawn at random from the combined population. The prior odds that the individual belongs to G_1 are $\pi_1/(1 - \pi_1)$. Show that the posterior odds given $\mathbf{z}^*$ are

$$\text{odds}(Y = 1 \mid \mathbf{z}^*) = \frac{\pi_1}{1 - \pi_1} \exp(\alpha + \beta^T \mathbf{z}^*),$$

   where the logistic regression coefficients are given by

$$\alpha = \frac{1}{2}\mu_2^T \boldsymbol{\Sigma}^{-1}\mu_2 - \frac{1}{2}\mu_1^T \boldsymbol{\Sigma}^{-1}\mu_1$$
$$\beta = \boldsymbol{\Sigma}^{-1}(\mu_1 - \mu_2).$$

4. Below you can see a description of the data collection in an **imaginary** study. The objective of the study was to find factors that have an impact on the choice of the brand of a new car. Your task is to write a paragraph that tells how the statistical analysis was carried out in the study.

    The questionnaire was sent to all private person who have bought a new car in March 2008. The addresses were obtained from The Finnish Vehicle Administration. 9799 questionnaires were sent and the response rate was 65 %. The data collected included the brand of the new car, the brand of the old car (if any), the year when the old car was bought, and age, sex, education (primary/secondary/university) and annual income (euros/year) of the owner.