Exercises 2

1. Let $y_1, \dots y_n$ be a random sample from $N(\mu, \sigma^2)$ where mean μ and variance σ^2 are unknown. Show that $\hat{\mu} = \bar{y} = \sum_{i=1}^n y_i/n$ and $\hat{\sigma}^2 = \sum_{i=1}^n (\bar{y} - y_i)/n$ maximize the likelihood function

$$L(\mu, \sigma^2) = \prod_{i=1}^n \left[\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y_i - \mu)^2}{2\sigma^2} \right) \right]$$

2. Let $y_1, \dots y_n$ be a random sample from $N(\mu, \sigma^2)$ where mean μ and variance σ^2 are unknown. Derive the maximum likelihood estimate for $\gamma = \mu/\sigma$.

3. Suppose that X_1 and X_2 are independent unit exponential random variables. Show that the distribution of $Y = \log(X_1/X_2)$ is

$$f_y = \frac{\exp(y)}{(1 + \exp(y))^2}$$

for $-\infty < y < \infty$.

4. Show that the logistic density

$$f(x) = \frac{\exp(x)}{(1 + \exp(x))^2}$$

is symmetrical about zero. Find the cumulative distribution function and show that the $100p$ percentile occurs at

$$x_p = \log(p/(1 - p))$$

5. Find a recent article where a GLM is used in data analysis. Identify the application area, the type of the GLM, the response variable, the explanatory variables and the sample size.