EVOLUTION AND THE THEORY OF GAMES
(sample exam)

1. Give the definition of a Nash equilibrium and calculate all Nash equilibria (pure and mixed) of the following game:

	y_1	y_2	y_3
x_1	2,0	0,1	1,2
x_2	0,2	1,0	2,1
x_3	-1,0	2,1	1,3

2. Give the definition of an evolutionarily stable strategy (ESS) and show that x_2 in the following game is the only ESS (pure or mixed):

	x_1	x_2
x_1	1 , 1	0 , 2
x_2	2 , 0	0 , 0

3. Show that the supports of two different mixed ESS-es of the same game with finitely many pure strategies cannot be nested (i.e., the support of one ESS cannot be a subset of the support of the other).

4. Calculate all ESS-es (pure and mixed) of the following game:

	y_1	y_2
x_1	1,2	2,3
x_2	2,1	1,0
x_3	1,3	0,2

5. Two males are competing for the same female, but it is the female who chooses the male. Each male can choose to advertise (A) or not to advertise (nA) himself. Think of buying her an expensive present or taking her on an expensive holiday trip.

There are two sub-games defined by which male is temporally the female's favorite: in sub-game Γ_r the row-player is favorite, while in Γ_c the column-player is favorite. There are thus four pure strategies for the males to choose from:

$$(A, A), (A, nA), (nA, A), (nA, nA)$$

where the first (resp. second) of each pair defines the action in the sub-game Γ_r (resp. Γ_c).

The female always chooses the male that advertises himself and at the same time is her favorite. If her favorite does not advertise himself, but the other male does, then she switches favorites but otherwise does not make a choice yet. If neither male advertises himself, nothing changes in the status of the males, but she also does not choose.

The value for a male of being chosen by the female is V, and the cost of advertising is C. The probability that there is a next round at all (i.e., the probability that she does not get bored and walk away, i.e., the *discounting factor*) is denoted by $\varepsilon \in (0,1)$. The payoff matrices (payoffs to the row-player!) of the respective sub-games thus are

Γ_r	A	nA
A	$V-C$	$V-C$
nA	$\varepsilon\Gamma_c$	$\varepsilon\Gamma_r$

Γ_c	A	nA
A	$-C$	$\varepsilon\Gamma_r$
nA	0	$\varepsilon\Gamma_c$

Calculate the payoff matrix for the full game $\Gamma = (\Gamma_r, \Gamma_c)$ assuming that both males have an equal probability of starting off as the female's favorite. Which of the four pure strategies is an ESS if $V > C$ and which if $V < C$?