8th lecture

6 Asymmetric games

Until now, the evolutionary games were symmetric in the sense that the players were interchangable.

This is often not the case:
- One player may be stronger than the other
- One may have arrived before the other
- Owner vs intruder
- male vs female
etc.

Such asymmetries may affect the payoffs or the strategy sets

What we need is an ESS solution concept where the players can play different strategies

Asymmetric ESS

Roles	X	Y
Strat. sets	$\mathbb{X}$	$\mathbb{Y}$

Payoff $x \in \mathbb{X}$ against $y \in \mathbb{Y}$ is $K_x(x, y)$
" $y \in \mathbb{X}$ " $x \in \mathbb{X}$ is $K_y(x, y)$

Idea: Use socalled conditional strategies to reformulate an asymmetric game as a symmetric one for which we already have an ESS concept, and then translate back to the asymmetric game

<u>Conditional strategy</u> $\binom{x}{y} \in \mathbb{X} \times \mathbb{Y}$:

"play x if in role X; play y if in role Y"

- Conditional strategies presume that the players know one-another's role (i.e., "perfect knowledge")
- A conditional strategy is independent of the role. $\Longrightarrow$ We're back in the symmetric case again.
- Given any set of identical X×Y contests, the number of individuals in role X is equal to the number of individuals in role Y $\Longrightarrow$ The prob that a given individual is in a given role is 1/2

<u>Translation to symmetric game</u>.

$z_i = \binom{x_i}{y_i} \in \mathbb{X} \times \mathbb{Y}$ $(i=1,2)$ conditional strategy.

$\Rightarrow$ $E(z_1, z_2) = \frac{1}{2} K_X(x_1, y_2) + \frac{1}{2} K_Y(x_2, y_1)$

Since this is a symmetric game,

$z^* = \binom{x^*}{y^*} \in \mathbb{X} \times \mathbb{Y}$ is ESS iff $\forall z = \binom{x}{y} \neq z^*$.

① $E(z, z^*) \leq E(z^*, z^*)$

and, in case of equality in ①,

② $E(z, z) < E(z^*, z)$

■.

By writing this out in terms of x and y we get:

Definition.

$(x^*, y^*) \in \mathbb{X} \times \mathbb{Y}$ is an ESS of the asymmetric $X \times Y$ if, $\forall (x,y) \neq (x^*, y^*)$,

① $K_x(x, y^*) + K_y(x^*, y) \leq K_x(x^*, y^*) + K_y(x^*, y^*)$

and, in case of an equality in ①,

② $K_x(x,y) + K_y(x,y) < K_x(x^*, y) + K_y(x, y^*)$

∎

Proposition. (with perfect knowledge).

(x^*, y^*) is an ESS of an asymmetric game if and only if it is a strict Nash equilibrium

Proof.

"$\Leftarrow$" Suppose (x^*, y^*) is a strict Nash equilibrium. Then

$$\begin{array}{lll} K_x(x, y^*) < K_x(x^*, y^*) & & \forall\, x \neq x^* \\ + \; K_y(x^*, y) < K_y(x^*, y^*) & & \forall\, y \neq y^* \end{array}$$

, 1st ESS cond

$\Rightarrow K_x(x, y^*) + K_y(x^*, y) < K_x(x^*, y^*) + K_y(x^*, y^*)$ ⊛
for all $x \neq x^*$ and all $y \neq y^*$

(Subst. of $x = x^*$, $y \neq y^*$ or $x \neq x^*$, $y = y^*$ gives back the strict Nash condition.)

So, ⊛ is true for all $(x, y) \neq (x^*, y^*)$.

"$\Longrightarrow$" Suppose (x^*, y^*) is an ESS of an asymmetric game.

We first show that (x^*, y^*) is a Nash equilibrium.

To this end let $x \neq x^*$ and $y = y^*$. Then from ① (in definition p. 94)

$$\boxed{K_x(x, y^*) \leq K_x(x^*, y^*)}$$

Likewise, for $x = x^*$ but $y \neq y^*$

$$\boxed{K_y(x^*, y) \leq K_y(x^*, y^*)}$$

In other words, (x^*, y^*) is Nash (p. 86).

We now show that (x^*, y^*) is a strict Nash equilibrium.

To reach a contradiction, suppose (x^*, y^*) is not a strict Nash equil.

Then $\exists x_0 \neq x^* : K_x(x_0, y^*) = K_x(x^*, y^*)$

or $\exists y_0 \neq y^* : K_y(x^*, y_0) = K_y(x^*, y^*)$

Without loss of generality assume the first is the case

Then for $(x, y) = (x_0, y^*)$ we have

$$K_x(x, y^*) + K_y(x^*, y) = K_x(x_0, y^*) + K_y(x^*, y^*) =$$

$$= K_x(x^*, y^*) + K_y(x^*, y^*)$$

So, condition ① (p. 94) is satisfied with equality

Hence, (x^*, y^*) being ESS, the second condition ② must be satisfied as well, i.e.,

$$K_x(x_0,y^*) + K_y(x_0,y^*) < K_x(x^*,y^*) + K_y(x_0,y^*)$$

$$\iff \cancel{K_x(x^*,y^*)} + K_y(x_0,y^*) < \cancel{K_x(x^*,y^*)} + K_y(x_0,y^*)$$

$$\iff K_y(x_0,y^*) < K_y(x_0,y^*)$$

which is a contradiction.

∎

Corollary.

An ESS in an asymmetric game always consists of pure strategies

Proof.

A Nash equilibrium in which one of the strategies is a mixed strategy is not a <u>strict</u> Nash equilibrium.

(see p. 40)

∎

If (x^*, y^*) is mixed Nash, then every pure strat' in the support of x^* has the same payoff as x^*

Example (Asymmetric H-D game)

X : arrived first. ("owner")
Y : arrived second. ("intruder")

		Y: H	Y: D
X:	H	$\frac{V-C}{2}, \frac{V-C}{2}$	$V, 0$
	D	$0, V$	$\frac{V}{2}, \frac{V}{2}$

- If $V > C$, then (H,H) is a strict Nash equilibrium, and therefore is an ESS.
- If $V = C$, then (H,H) is a Nash equilibrium, but not a strict Nash equilibrium, and therefore is not an ESS.
- If $V < C$, then (H,D) and (D,H) are both strict Nash equilibria and therefore ESS.
- There never are mixed ESSs in an asymmetric game.

Remarks:

- The payoff matrix is symmetric. The asymmetry is in the roles (the roles are arbitrary but must be known)
- If $V < C$, then at the ESSs (H,D) or (D,H), no escalated fights occur: The contest is settled by convention.

Association of action with role is ESS

- an arbitrary asymmetry can settle the conflict, provided the asymmetry is perceived by both players.

Biological example

Spider: Oecibus civitas

asymmetry: Web intruder (X)
Web owner (Y)

(H, D) appears to be the ESS that has become established i.e., the site owner immediately gives away his web to an intruder.

(H, D): "paradoxical"

(D, H): "Bourgois"

"Paradoxical" is rarely found in nature (exception Oecibus civitas)

Why is this?

a necessary condition for "paradoxical" to be an ESS is that $V < C$, so maybe resources are too valuable in nature to satisfy this condition.

Other explanation: The resource has different values for the resource owner and the non-owner.

→ Next example.

Example.

Resource = territory

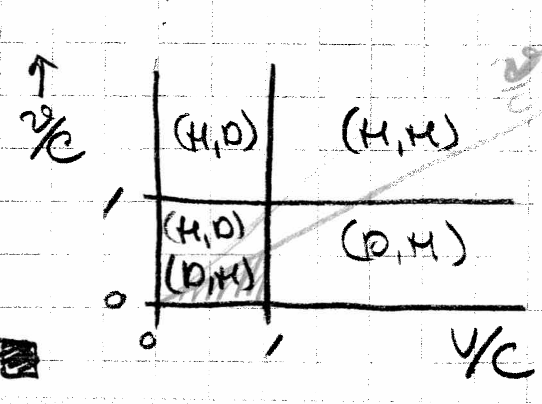
X = intruder
Y = owner

V = value of territory to owner
v = value of territory to intruder, which may be less than V, because the intruder still has to pay the cost of learning its territory by spending time on exploration, increasing risk of being preyed upon, e.g.

Y:

X:	H	D
H	$\frac{v-C}{2}, \frac{V-C}{2}$	$v, 0$
D	$0, V$	$\frac{1}{2}v, \frac{1}{2}V$

- (H,H) is ESS $\iff$ $v/C \geq 1$ & $V/C > 1$
- (D,D) is never ESS
- (H,D) is ESS $\iff$ $V/C < 1$
- (D,H) is ESS $\iff$ $v/C < 1$



If $v \ll V$, then "paradoxical" (H,D) is ESS in only very small part of the feasible parameter space