

EVOLUTION AND THE THEORY OF GAMES (Spring 2009)
EXERCISES 16 - 19

16. Generalize the Bishop-Cannings theorem: Let $\mathbb{X} \subset \mathbb{R}$ be an interval, and let $F : \mathbb{X} \rightarrow \mathbb{R}$ be a distribution function representing a mixed strategy over \mathbb{X} with support S . Show that if F is an ESS, then $E(x, F) = E(F, F)$ for every $x \in S$ where $E(x, F)$ is continuous as a function of x .
17. Let F and G be two different distribution functions on a given interval $\mathbb{X} \subset \mathbb{R}$ with supports S_F and S_G , respectively. Show that if F and G are ESSs, then S_F cannot be a subset of S_G and *vice versa*.
18. Analyze the War of Attrition (WoA) where the cost of displaying is a strictly increasing but otherwise arbitrary function of time .
19. Two lions are stalking the same prey. Both start at the same distance from the prey and move at the same speed. When one of the lions attacks, the other follows almost immediately. During the attack the lions chase the prey who runs away but inevitably is captured. The lion who attacked first has a somewhat higher probability of capturing the prey than the one that followed. The two lions are from different packs and do not share the prey.

We can model this situation as follows: Let $x, y \in [0, 1]$ denote the attack distances for the two lions, i.e., the lions start stalking at one unit distance from the prey and plan to attack at a distance x or y , respectively. The actual attack distance is given by the smallest of the two numbers, because the other lion follows almost immediately after the first initiates its attack. The payoff to strategy y against strategy x then is given by

$$E(y, x) = \begin{cases} pV - cx & \text{if } y < x \\ \frac{1}{2}V - cx & \text{if } y = x \\ (1 - p)V - cy & \text{if } y > x \end{cases} \quad (1)$$

where $0 < p < \frac{1}{2}$ is the probability that the lion who follows the first attack gets the prey, V the value of the prey, and cx the cost of the pursuit if the attack was initialized at a distance x from the prey.

Analyze this game along the same lines as in the War of Attrition.