University of Helsinki / Department of Mathematics and Statistics
SCIENTIFIC COMPUTING
Exercise 12, 7.12.2009

N.B. The files mentioned in the exercises (if any) are available on the course homepage.

1. Show experimentally that for real 2×2 matrices `A=[a b; c d]` the following equality holds;

$$cond(A) = s + \sqrt{s^2 - 1} \quad \text{where} \quad s = (a^2 + b^2 + c^2 + d^2)/(2|\det(A)|).$$

2. Use the data d122dat.dat to fit the model $f(\lambda_1, \lambda_2, \lambda_3, x) = \lambda_1/(1 + (x - \lambda_2)^2) + 1/(1 + (x - \lambda_3)^2)$. Use the initial values `[1,-1,2]` as a guess. Hint: `parfit` or `parf04`.

3. An earlier version of the program e916.m contained a small error: the exact solution is

```
% N.B. The term   c2*one./(t.^2) is now OK,
exact = (1.1-c2)*t +c2*one./(t.^2) - 0.3*sin(log(t)) - 0.1*cos(log(t));
```

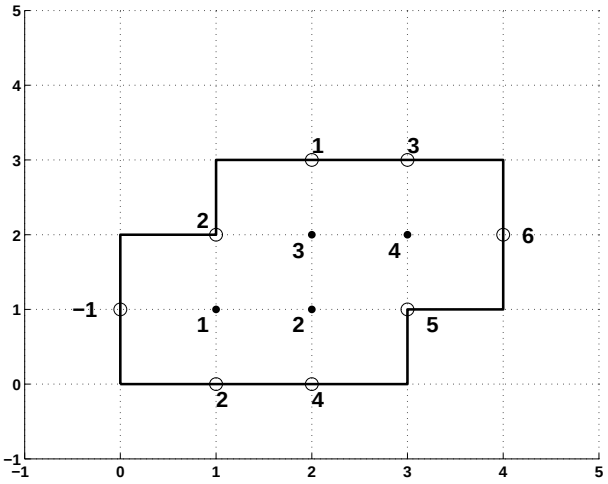
Correct the program, if necessary, and modify it so that it runs for the step size $(b - a)/n, n = 8 : 20$. Print the maximum error for each value of n.

4. Solve Dirichlet's problem

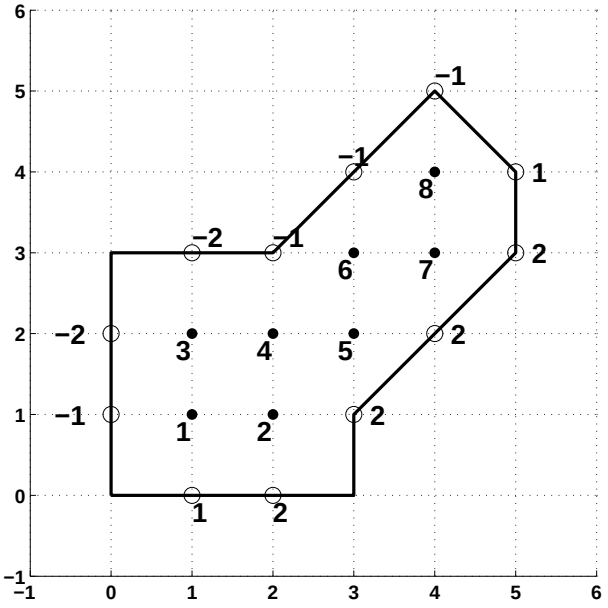
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

in the situation pictured below, by using the boundary values and the numbering of variables as in the picture. The sidelength of a square is 1.

FILE: ~/mme07/teht/d12/d12.tex — 30. marraskuuta 2009 (klo 8.21).



5. As 4, but in the situation of the following picture.



6. Consider the data $(x_j, y_j), j = 1, ..., m$, and set

$$f(a,b,c,d,x) = ax^2 + bx + c + d/x\,, \quad S = \sum_{j=1}^{m} (y_j - f(a,b,c,d,x_j))^2\,.$$

A researcher is modelling the political awareness in EU countries using this model.

(a) Help the researcher to set up the normal equations. (Recall that these are $\frac{\partial S}{\partial a} = 0$, $\frac{\partial S}{\partial b} = 0$, $\frac{\partial S}{\partial c} = 0$, $\frac{\partial S}{\partial d} = 0$.) Do not solve the normal equations.

(b) Use the method of problem d105 to write the problem in matrix form $X\lambda = Y$, where $X(j,:) = [x_j^2, x_j, 1, 1/x_j], Y(j,1) = y_j$ and $\lambda = [a; b; c; d]$. Then generate synthetic data and use solve this system of equations $\lambda = X \setminus Y$.