

University of Helsinki / Department of Mathematics and Statistics  
SCIENTIFIC COMPUTING  
Exercise 11, 30.11.2009

**N.B.** The files mentioned in the exercises (if any) are available on the course homepage.

1. The Gram–Schmidt method also applies to orthogonalize a system of functions. Use this method to orthogonalize the system  $\{1, x, x^2, x^4\}$ . of the space  $C([-1, 1])$  with the inner product  $(f, g) = \int_{-1}^1 f(x)g(x) dx$ .

2. Part (b) of this problem deals with the so called Gibbs phenomenon.

- (a) Show that for each fixed  $x$  the number  $S_n(x) = \frac{nx}{1+n^2x^2}$  approaches zero when  $n$  grows to  $\infty$ , and find the extremum values of  $S_n(x)$  with respect to  $x$ . Graph the function  $S_n(x)$  when  $n=2:2:10$ .
- (b) Show that the Fourier series of  $f(x) = (\pi-x)/2$  on  $(0, 2\pi)$  is  $\lim_{n \rightarrow \infty} S_n(x)$ , where  $S_n(x) = \sum_{k=1}^n \frac{1}{k} \sin(kx)$ . Graph  $S_n$  for  $n=10:2:20$  and find graphically the global maximum  $x_0 \in (0, 2\pi)$  of  $S_n(x)$  and estimate graphically the number  $|S_n(x_0) - f(x_0)|/|f(x_0)|$ .

Recall the Fourier series of a continuous function  $f : [0, 2\pi] \rightarrow \mathbf{R}$

$$\frac{1}{2}a_0 + \sum_{n=1}^{\infty} (a_n \cos(nx) + b_n \sin(nx)),$$

where

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos(nx) dx, \quad n = 0, 1, 2, \dots$$

and

$$b_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin(nx) dx, \quad n = 1, 2, 3, \dots$$

3. Find the  $n$ th partial sum of the Fourier series of  $f(x) = x^2$  on  $(0 < x < 2\pi)$  and graph it for  $n=4:2:10$ .

4. Consider again the problem of fitting a "line with a break point" to a data set, as in d101. Now, instead of choosing the break point  $(s, t)$  with a mouse click as we did in d101, use the method of the program parfit to find the best break point  $(s, t) = (\lambda_1, \lambda_2)$ . The object function will be, with

the notation of the solution to d101, s1+s2. Apply this optimized version of d101 to the data of d101. Recall that the object function value obtained in d101, after the fitting was 2.62. Do you get a better value this time?

5. An astronomer has the following observations about a comet approaching the Earth.

Taulukko 1: Comet coordinates

|   |      |      |      |      |      |      |      |      |      |      |
|---|------|------|------|------|------|------|------|------|------|------|
| x | 1.02 | 0.95 | 0.87 | 0.77 | 0.67 | 0.56 | 0.44 | 0.30 | 0.16 | 0.01 |
| y | 0.39 | 0.32 | 0.27 | 0.22 | 0.18 | 0.15 | 0.13 | 0.12 | 0.13 | 0.15 |

Determine the equation of the comet on the basis of this data using a quadratic function

$$ay^2 + bxy + cx + dy + e = x^2.$$

Hint: The problem yields the overdetermined system

$$ay_i^2 + bx_iy_i + cx_i + dy_i + e = x_i^2, i = 1, \dots, 10,$$

which we will solve with the LSQ method for the coefficient vector  $sol = (a, b, c, d, e)^T$ . We rewrite this as  $M * sol = w$  and its solution is obtained with  $sol = M \backslash w$  (or, alternatively,  $sol = pinv(M) * w$ ).

6. Write a program numdf, which computes the numerical derivative of a function at the points in a given vector, using the function number. The program call should be of the form

```
numdf('myf(x)',z, 1e-4)
```

where  $z = 0:0.05:1;$ , and myf is a function. Plot the error of the numerical derivation using the command `pic('cos(x)- numdf('sin(x)', x, 1e-4)')`. Hint: The file hlp116.m contains number and pic.