University of Helsinki / Department of Mathematics and Statistics
SCIENTIFIC COMPUTING
Exercise 07, 2.11.2009

`Problem sessions will be held on Monday at 16-18, B322.`

N.B. The files mentioned in the exercises (if any) are available on the course homepage

1. Use getpts to draw a triangle and generate random points in a polygon. Compute the mean of the random points and mark it on the polygon. Also compute the mean value of the points in the triangle. Hint: `help inpolygon`.

2. Consider functions $f(z) = \sum_{n=0}^{\infty} a_n z^n$ with complex coefficients a_n that satisfy $|f(z)| \leq 1$ for all $z, |z| \leq 1$. For instance, if a sequence (a_n) of complex numbers with $S = \sum_{n=0} |a_n| < \infty$ is given, then the function $f(z) = \sum_{n=0}^{\infty} b_n z^n$, $b_n = a_n/S$ has this property.

(a) Generate polynomial functions with complex coefficients ($a_n = 0$ for all large n) and plot image of the circle $\{z : |z| = r\}$ when $r \in (0, 1)$. Mark the point on this circle where $\sup\{|f(z)| : |z| \leq r\}$ is attained. Observe: Usually this point is not on the real positive real axis. Can you confirm this with your experiment?

(b) An inequality due to Bohr (Lond. M. S. Proc. (2) 13 (1913), 1-5) states that under the above hypotheses, $\sum_{n=0}^{\infty} |a_n| r^n \leq 1$ for all $r \in (0, 1/3]$. Verify this statement.

3. Recall the function $\text{erf}(x)$ studied in Problem 1/Exercise 5. Utilize it to express the function $P(x)$ in terms of $\text{erf}(x)$ and to tabulate the values $P(x), x = 0 : 0.2 : 2$ when $P(x) = \int_{-\infty}^{x} Z(t)dt$ and $Z(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$. You may want to look at the table 26.1 on p. 966 of Abramowitz-Stegun `http://www.math.sfu.ca/~cbm/aands/`, which contains these values.

4. Recall from linear algebra that $[a, b; c, d]^{-1} = T[d - b; -c\, a]$ if $1/T = ad - bc \neq 0$. Recall also that the Newton method for solving $h(w) = 0, h : \mathbb{R}^2 \to \mathbb{R}^2$ is the iteration $w_{n+1} = w_n - J_h(w_n)^{-1} h(w_n), n = 0, 1, 2, 3, ...$

a) Utilize the linear algebra formula to show that the Newton method for solving $f(x, y) = 0, g(x, y) = 0$ yields the sequence ($f_1 = \partial f/\partial x, f_2 =$

FILE: ~/MME08/demo/d07/d07.tex — 28. lokakuuta 2009 (klo 12.27).

$\partial f/\partial y$)

$$x_{n+1} = x_n - \frac{fg_2 - gf_2}{f_1g_2 - g_1f_2}, \quad y_{n+1} = y_n - \frac{f_1g - g_1f}{f_1g_2 - g_1f_2}.$$

(b) Apply this method when $f(x,y) = x^2 + y^2 - 1, g(x,y) = y - e^x$ with $x_0 = -0.9, y_0 = 0.2$.

5. Use parfit.m or parf04.m (instead of the algorithm of Problem 2 in Exercise 6) to fit the model

$$y = \lambda_1 + \lambda_2 * \sin(2 * \pi * (x - \lambda_3)/24))$$

to the data from Problem 2 of Exercise 6

```
x   0.0  2.0  4.0  6.0  8.0 10.0 12.0 14.0 16.0 18.0 20.0 22.0 24.0
y   6.3  4.0  6.6 10.9 14.6 19.1 24.3 25.7 22.9 19.5 15.9 10.3  5.4
```

6. Familiarize yourself with the content of the MATLAB help information for the commands griddata and contour. Use the corresponding examples from the MATLAB helpdesk to graph the temperature over Finland, given the x,y, coordinates and the temperatures in the following cities

```
nimi=str2mat('Turku','Tre','Hki','Oulu', 'Jkyla','Mikkeli', 'Vaasa');
x= [0 100 160    40 160 400   0];
y= [0  80   0   500 200 100 300];
t= [10  8  10     5   5   3   7];
```

Use this data and the function interp2 to obtain by interpolation the temperature in Salo (x=50, y=0).