University of Helsinki / Department of Mathematics and Statistics
SCIENTIFIC COMPUTING
Exercise 6, 19.10.2009

`Problem sessions will be held on Monday at 16-18, B322.`

N.B. The files mentioned in the exercises (if any) are available on the course homepage `http://mathstat.helsinki.fi/~vuorinen/mme07/`

1. The Senior Researcher studies the intelligence quotient (IQ) of the Ubuntu tribe in the Third World. The results of an IQ test are listed here:

IQ -range	# of samples	Normalized samples
61-70	89	
71-80	106	
81-90	84	
91 -100	94	
101-110	35	
111-120	23	
121-130	11	
131-140	1	
141-150	2	

Compute the mean μ and the variance σ^2 of the sample. Fill in the third column, for each row the normalized sample is # of samples divided by the total number of samples.

2. The daily temperature data is observed and the results appear in the table below. Create a file with these thirteen (x,y) pairs of this temperature data.

x	0.0	2.0	4.0	6.0	8.0	10.0	12.0	14.0	16.0	18.0	20.0	22.0	24.0
y	6.3	4.0	6.6	10.9	14.6	19.1	24.3	25.7	22.9	19.5	15.9	10.3	5.4

Let $m = \min\{y\}$, $M = \max\{y\}$ and set $a = 0.5*(M+m)$, $b = 0.5*(M-m)$. Try to find some reasonable integer value for the parameter c in the interval $[0, 24]$ so that the curve $y = a + b * \sin(2 * \pi * (x - c)/24))$ becomes as close to the data as possible. Carry out the following steps:

(a) Read the data (x_j, y_j), $j = 1, ..., 13$, from the file [or copy these values in a vector] and compute the maximum and minimum temperatures M and m . Then compute a and b.

FILE: ~/MME07/demo/d06/d06.tex — 12. lokakuuta 2009 (klo 13.18).

(b) For each $c = 0:24$ compute

$$A(c) = \sum_{j=1}^{13} (y_j - (a + b * \sin(2 * \pi * (x_j - c)/24)))^2 \,,$$

and choose the value of c that yields the minimal $A(c)$.

(c) With these values of the parameters a, b, c plot the curve $y = a + b * \sin(2 * \pi * (x - c)/24))$ and the data points in the same picture.

3. To fit a circle (1) $(x - c_1)^2 + (y - c_2)^2 = r^2$ to n sample pairs of coordinates $(x_k, y_k), k = 1, ..., n$ we must determine the center (c_1, c_2) and the radius r. Now (1) $\Leftrightarrow$ (2) $2xc_1 + 2yc_2 + (r^2 - c_1^2 - c_2^2) = x^2 + y^2$. If we set $c_3 = r^2 - c_1^2 - c_2^2$, then the equation takes the form

$$2xc_1 + 2yc_2 + c_3 = x^2 + y^2 \,.$$

Substituting each data point we get

$$\begin{bmatrix} 2x_1 & 2y_1 & 1 \\ & \vdots & \\ 2x_n & 2y_n & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} x_1^2 + y_1^2 \\ \vdots \\ x_n^2 + y_n^2 \end{bmatrix}$$

This system can be solved in the usual way for `c = matrix/ rhs` . Then $r = \sqrt{c_3 + c_1^2 + c_2^2}$. Apply this algorithm for the points generated by

```
r=0.5+0.5*rand(10,1);
theta=2*pi*rand(10,1);
clear x
clear y
x=3*r.*cos(theta) ;
y=3*r.*sin(theta);
```

Plot the data and the circle.

4. The number of participants of the weekly problem sessions of a mathematics course during the first six weeks were 21, 24, 17, 21, 14 and 17, respectively. Fit a model of the type

$$y = \lambda_1 \exp(-\lambda_2 x)$$

to this data and predict the number of participants in the 12th problem session.
Hint: It may (or may not) be a good idea to make a linear transform

$y' = y/25$, $x' = x/12$ for the fitting, and then use the program parfit.m/Lectures/Section 2 and finally to transform back to the original variables.

5. Familiarize yourself with the program `getpts.m` and use it to plot a closed polygon in the plane. Compute also its area with `polyarea`.

6. Consider the tabulated values `x=0:0.2:3.2; y=d071f(c,d,x)` of the function $d071f(x) = \sum_{j=1}^{m} c_j \sin(d_j * x)$ with `c=[1 2 3 2 1]`, `d=[3 2 1 2 2]` . The data is interpolated to the points `x=0.0:0.05:3.2` by using two different methods; (a) `interp1` , (b) `spline`. Find the maximum error of each method by comparing the interpolation to the values of the function at these points.