University of Helsinki / Department of Mathematics and Statistics
SCIENTIFIC COMPUTING
Exercise 01, 14.9.2009

`Problem sessions will be held on Monday at 16-18, C128.`

N.B. The files mentioned in the exercises (if any) are available on the course homepage
`http://wiki.helsinki.fi/display/mathstatKurssit/Syksy+2009`

1. Apply the recursion formula $x_0 = 1, x_{n+1} = \frac{1}{2}(x_n + \frac{a}{x_n}), n = 0, 1, 2, \ldots$ for $\sqrt{a}$ to compute $\sqrt{3}$. Print the results in the following format:

```
n  x(n)   Error
0  1
.....
6  ...
```

2. Approximations to the number π are given by the formula

$$p(n) = \sum_{k=0}^{n} \frac{1}{16^k} \left(\frac{4}{8k+1} - \frac{2}{8k+4} - \frac{1}{8k+5} - \frac{1}{8k+6} \right).$$

Print the first few results in the same format as in problem 1.

3. In Solmu 2/2005 (`http://solmu.math.helsinki.fi/2005/2/`) the following problem was studied. Is it true that a continuous function $f : (0, \infty) \to (0, \infty)$ satisfying the conditions:

1. $f(2x) = 2f(x)$, and
2. $f(1) = c$

is always of the form $f(x) = cx$. In the article, the following counterexample was presented:

$$f(x) = 2^{-n}x^2 + 2^{n+1} \text{ for } x \in [2^n, 2^{n+1}),$$

where $n = 0, \pm 1, \pm 2, \ldots$. Plot the graph of this function.

FILE: ~/mme09/demo09/d01/d01.tex — 8. syyskuuta 2009 (klo 7.13).

4. Let $(x_j, y_j), j = 0, 1, ..., n$ be the vertices of a polygon with $(x_0, y_0) = (x_n, y_n)$. The area of the polygon is given by $a = \frac{1}{2}\sum_{i=1}^{n} t_i$ with $t_i = x_{i-1}y_i - x_i y_{i-1}$. Carry out the following steps for each of the regular polygons triangle, square and hexagon:

(a) Choose vertices and compute the area by school geometry.

(b) Compute the area by the formula and compare to the exact value.

(c) Plot the figure.

5. Hilbert's inequality says that for $a_k, b_k \geq 0$

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{a_m b_n}{m+n+1} \leq \pi(\sum_{m=0}^{\infty} a_m^2)^{1/2}(\sum_{n=0}^{\infty} b_n^2)^{1/2}.$$

Carry out a numerical verification of this inequality.

6. Consider a data set $(x_i, y_i), i = 1, ..., n$. We define $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ and

$$\begin{aligned} ss_{xx} &= \sum(x_i - \overline{x})^2 &&= \sum x^2 - n\overline{x}^2 \\ ss_{yy} &= \sum(y_i - \overline{y})^2 &&= \sum y^2 - n\overline{y}^2 \\ ss_{xy} &= \sum(x_i - \overline{x})(y_i - \overline{y}) &&= \sum xy - n\overline{xy}. \end{aligned}$$

Write a MATLAB program that computes the correlation coefficient r of the data set, defined as

$$r^2 = \frac{ss_{xy}^2}{ss_{xx}ss_{yy}}.$$

Create a syntetic data $x_i = i * 0.1, y_i = 0.7 * x_i + c * error_i$ where $error$ is uniformly distributed in $(-0.1, 0.1)$ with mean 0. One expects that the correlation coefficient decreases when c increases from 0.5 to 1. Check this with MATLAB.